UNIT - I

AMPLITUDE MODULATION.

I Review of signals & system

Signal:

* Physical Quantity that varies with time, space or another independent variable. Anything that carries information is called signal.

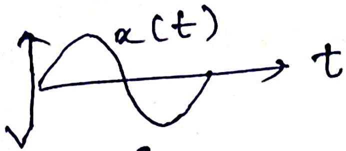
System:-

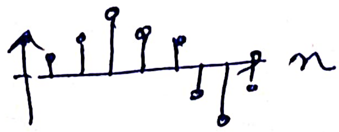
I/p signal → System → O/p signal

* As a physical device that generates a response or an o/p signal for a given i/p signal.

classification of signals:-

→ one dimensional → one variable
→ Two dimensional → more than one variable
→ Three dimensional →

Continuous Time Signal → 

Discrete Time Signal → 

1) Continuous time signal → i/p & o/p varying continuously with respect to time. $x(t)$ → System → $y(t)$

2) Discrete time signal /system. o/p depends on i/p, with instant time interval

$x(n)$ → System → $y(n)$.

Adv:-
* Greater accuracy
* Ease of storage
* Speed of Processing
* Flexibility

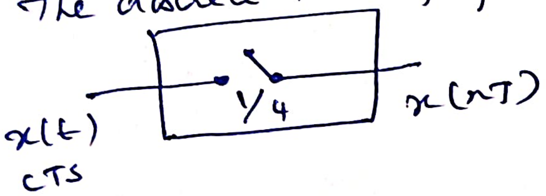
Application :- * Telecommunication
* Defence
* Medicine.

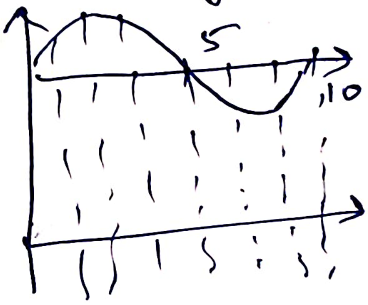
Concepts of frequencies in Analog & Digital Signals :-

i) Analog signals :-
Signal varying continuously with respect to time called Analog signal.

$x(t) = A \sin(\Omega t + \theta)$ rad/sec.

ii) Discrete Time signals :-
Signal varying with constant interval of time.
The discrete time frequency is denoted by 'ω' (or) 'f'.





$x(n) = A \sin(2\pi F nT + \theta)$ $\quad \left[\because F_s = \frac{1}{T}, T = \frac{1}{F_s}\right]$

$x(n) = A \sin\left(2\pi \frac{F}{F_s} n + \theta\right)$ $\quad [\because 2\pi F = \Omega]$

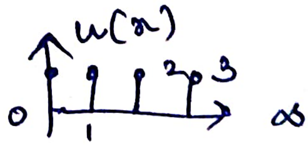
$$\boxed{\omega = 2\pi \frac{F}{F_s}}$$

Frequency Ranges : → $-\infty < F < \infty$ → infinite in nature
→ $-\pi < f < \pi$ & $-\pi < \omega < \pi$.

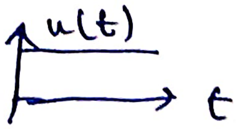
Elementary Signal / Sequence :-

1) unit step sequence / signal :-

$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$

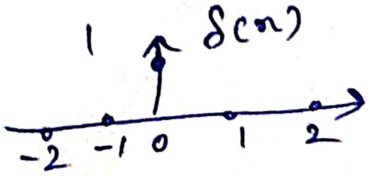


$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$

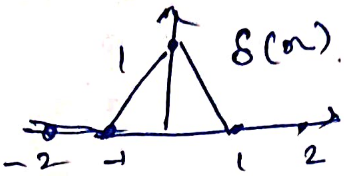


2) unit impulse Sequence :-

$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$

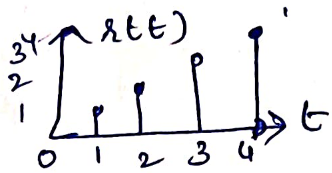


$$\delta(t) = \begin{cases} 1, & t=0 \\ 0, & t\neq 0 \end{cases}$$

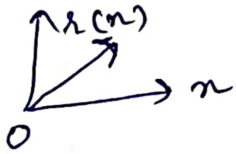


3) Ramp sequence:-

$$r(n) = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

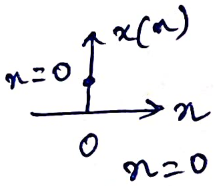


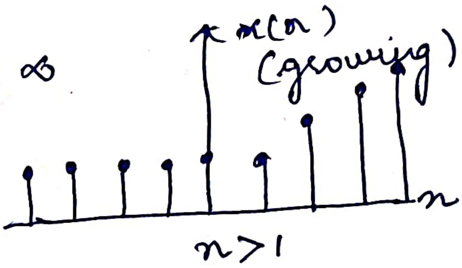
$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



4) Exponential signal:-

$$x(n) = a^n \quad -\infty \leq n \leq \infty$$





5) Complex exponential Signal:-

$$x(n) = a^{\sigma \pm j\Omega n}.$$

Classification of Discrete time signal:-

1. Periodic & Aperiodic signal:-

$$x(n+N) = x(n) \forall n.$$

$$\boxed{x(n) = A \sin(\omega n + \theta)}$$

2. Energy & power signal:-

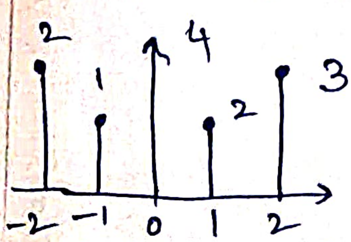
$$E = \sum_{n=N}^{N} |x(n)|^2, \quad -\infty < n < \infty$$

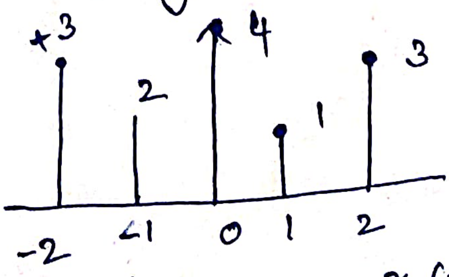
$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2.$$

3. Even & odd signal:-

$x(-n) = x(n) \; \forall n \rightarrow$ even signal.

$x(-n) = x(n) \forall n \rightarrow$ odd signal.





$x(n)$ $\quad x_e(n) = \dfrac{x(n) + x(-n)}{2}$ $\quad x_o(n) = \dfrac{x(n) - x(-n)}{2}$

ii) Causal & Non-Causal Signal :-

$x(n) = 0 \ , n < 0.$

II Time & Frequency Domain Representation of signal :-

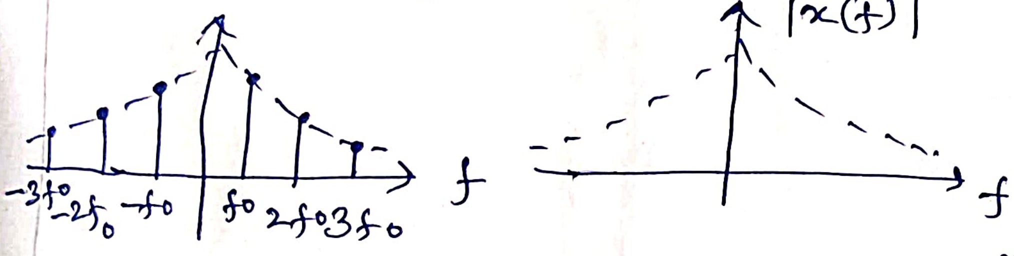
* Electrical signal represented — Voltage signal / current signal.

1) Time domain Representation :-

Fourier Transformation :-

Periodic Signals that are extended over interval $(-\infty, \infty)$

Non-Periodic signals $\rightarrow -\infty$ to ∞



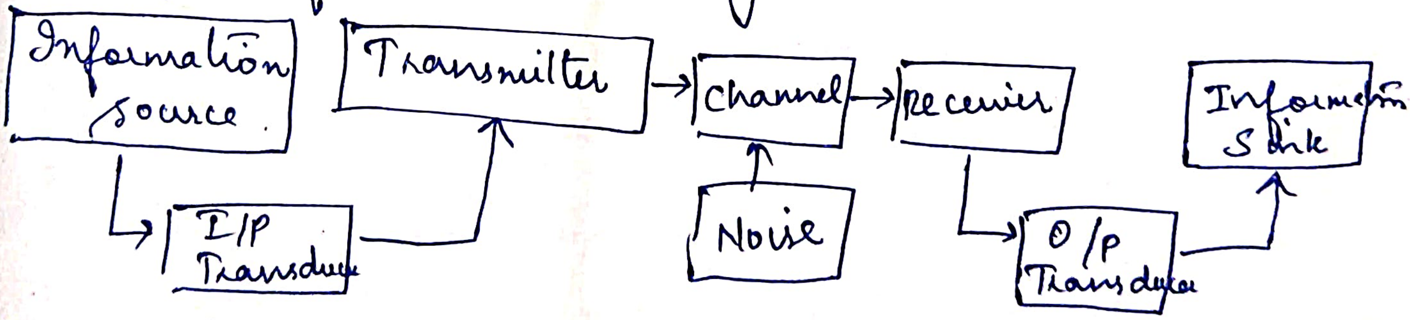
Fourier Transform $x(w) = F[x(t)] = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt.$

Inverse Fourier Transform

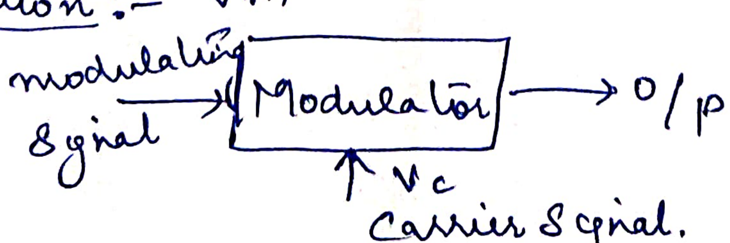
$$F^{-1}[X(W)] = x(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} X(W)\, e^{jwt}\, dW.$$

Introduction To Communication system :-

The Process of establishing connection (or) link b/w two points for information exchange.



Modulation :-



Process by which some characteristics of a carrier wave (signal) is varied in accordance with a modulating wave (signal)

III Principles of AM systems:-

* AM is defined as the process, in which the max. amplitude of carrier wave (signal) is proportional varied in accordance to instantaneous value (amplitude) of modulating (or) baseband signal, frequency & Phase remain constant.

Modulating Signal $E_m(t) = E_m \sin \omega_m t$

Carrier Signal $E_c(t) = E_c \sin \omega_c$

AM signal $E_{AM}(t) = E_{AM} \sin \omega_c t$.

After Modulation $E_{AM} = E_c + E_m(t)$

$$= E_c + E_m \sin \omega_m t$$

$$= E_c \left[1 + \frac{E_m}{E_c} \sin \omega_m t\right]$$

$$= E_c \left[1 + m_a \sin \omega_m t\right] \quad \left[m_a = \frac{E_m}{E_c}\right]$$

AM wave $\boxed{E_{AM}(t) = E_c\left[1 + m_a \sin \omega_m t\right] \cdot \sin \omega_c t}$

1) → Modulation Index :- (modulation co-efficient or degree of modulation)

$$m_a = \frac{E_m}{E_c} = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}$$

i) when $E_m \leq E_c$ → m values between 0 to 1

ii) when $E_m > E_c$ → modulation index > 1.

2) Percentage of Modulation :-

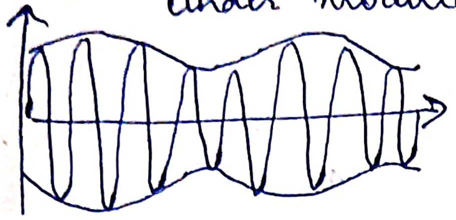
Modulation index in % is called % of Modulation

$$\% \, m_a = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} \times 100$$

modulating signal $m_a = \sqrt{m_1^2 + m_2^2 + m_3^2 + \cdots}$

Degrees of modulation :-

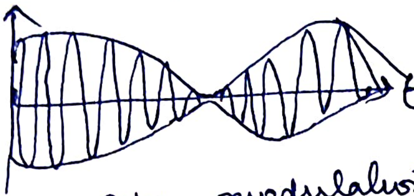
under modulation



$m_a < 1$, $E_m < E_c$

* AM Signal does not reach zero amplitude axes

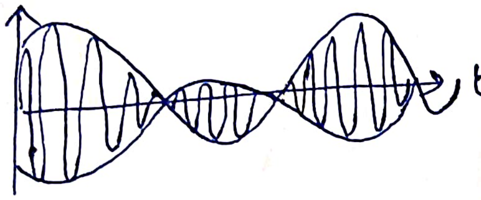
critical modulation



$m_a = 1$, $E_m = E_c$

* AM reaches zero amp. axis

over modulation



$m_a > 1$, $E_m > E_c$

* Envelope crosses the zero amp.

3) Power of single Tone AM signal.

Carrier signal $= A \cos w_c t$

modulating signal $= x(t) = V_m \cos w_m t$.

Then unmodulated or carrier power,

$$P_c = \overline{(A \cos w_c t)^2} = \frac{A^2}{2} \text{ (or) } \frac{E_c^2}{2}$$

Side band power.

$$P_s = \frac{1}{2}\overline{x^2(t)} = \frac{1}{2}\overline{(V_m \cos w_m t)^2}$$

$$= \frac{1}{2} V_m^2/2 = \frac{1}{4} V_m^2$$

$\because$ $P_t = P_c + P_s$

$$= \frac{A^2}{2} + \frac{1}{4} V_m^2$$

$$\boxed{\therefore P_t = \frac{A^2}{2}\left[1 + \frac{1}{2}\left(\frac{V_m}{2}\right)^2\right]}$$

$\because \frac{V_m}{A} = \frac{E_m}{E_c}$

where

$$\frac{V_m}{A} = \frac{\text{max. baseband amp}}{\text{Max. Carrier Amp.}} = m_a \text{ (modulation Index)}$$

$$P_t = \frac{A^2}{2}\left[1 + \frac{1}{2} m_a^2\right] \quad \left[\because \frac{A^2}{2} = P_c\right]$$

$$\boxed{P_t = P_c\left(1 + \frac{m_a^2}{2}\right)}$$

multi-tone $P_t = P_c\left(1+\frac{m_1^2}{2}+\frac{m_2^2}{2}+\frac{m_3^2}{2}+\dots+\frac{m_n^2}{2}\right)$ (7)

Current Relations :- $P_t = P_c\left(1+\frac{m_a^2}{2}\right)$

$$I_t = I_c\sqrt{1+\frac{m_a^2}{2}}$$

4) **Spectrum of AM :-**

Sinusoidal carrier wave $c(t) = A_c\cos(2\pi f_c t)$

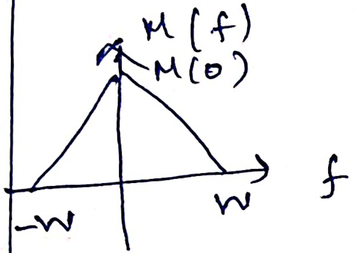
AM wave as function

$$s(t) = A_c[1+k_a m(t)]\cos(2\pi f_c t) \quad [\because k_a = \text{amplitude sensitivity}]$$

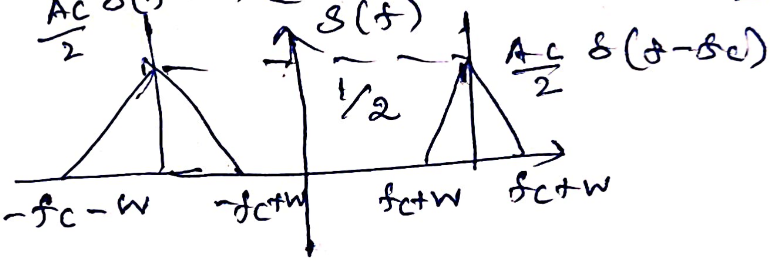
1. $k_a m(t) <$ unity.
2. f_c is $>$ than highest frequency W $[f_c \gg W]$

Fourier Transform,

$$S(f) = \frac{A_c}{2}[\delta(f-f_c)+\delta(f+f_c)] + \frac{k_a A_c}{2}[M(f-f_c)+M(f+f_c)]$$



Spectrum Base band signal



$$V_{AM}(t) = \underbrace{E_c\cos\omega_c t}_{\text{carrier}} + \frac{m_a E_c}{2}\underbrace{\cos(\omega_c-\omega_m)t}_{\text{LSB}} + \frac{m_a E_c}{2}\underbrace{\cos(\omega_c+\omega_m)t}_{\text{USB}}$$

5) **Transmission Efficiency :-**

$$\eta = \frac{P_{LSB}+P_{USB}}{P_t} = \frac{\frac{m^2}{4}P_c+\frac{m^2}{4}P_c}{\left(1+\frac{m^2}{2}\right)P_c} = \frac{m^2}{2+m^2}$$

% of Transmission efficiency,

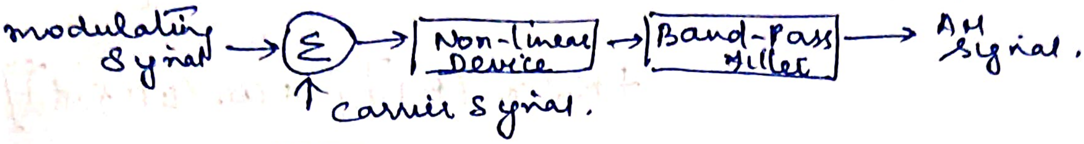
$$\eta = \frac{m^2}{2+m^2}\times 100\%$$

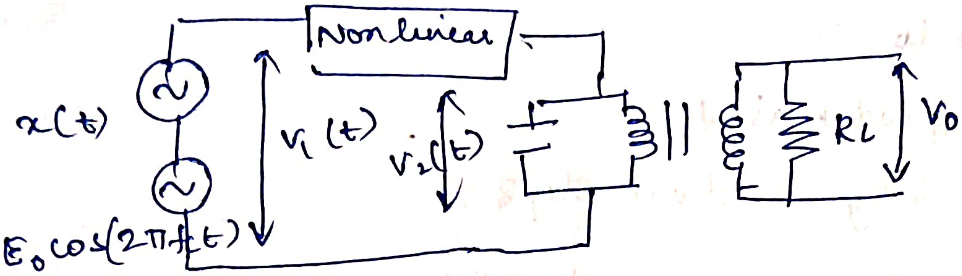
$$\boxed{\eta = \frac{m_a^2}{2+m_a^2}\times 100\%}$$

6) **Generation of AM Wave :-**

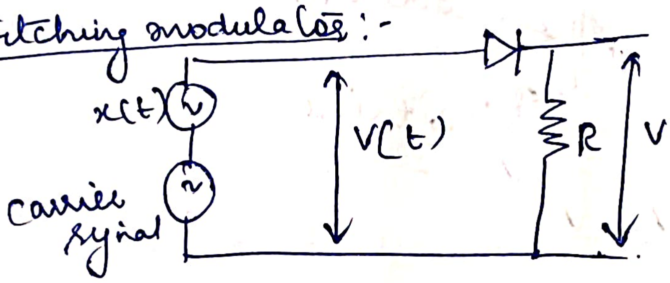
i) Square Law Modulator:

1. Modulating signal & Carrier Source 2. Non-linear Device
3. Band Pass Filter.



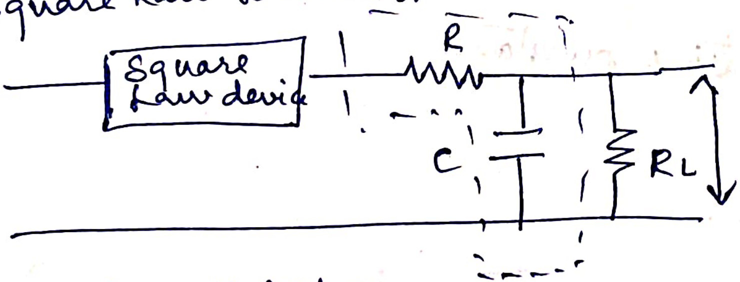


(ii) Switching modulator:-

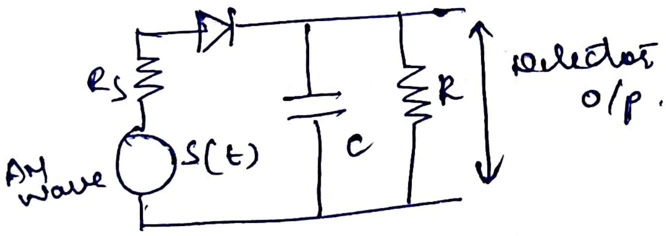


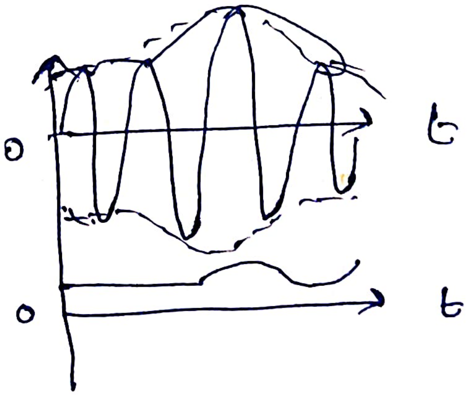
7) **Detection (Demodulation of AM wave):-**

1) Square Law Detector:



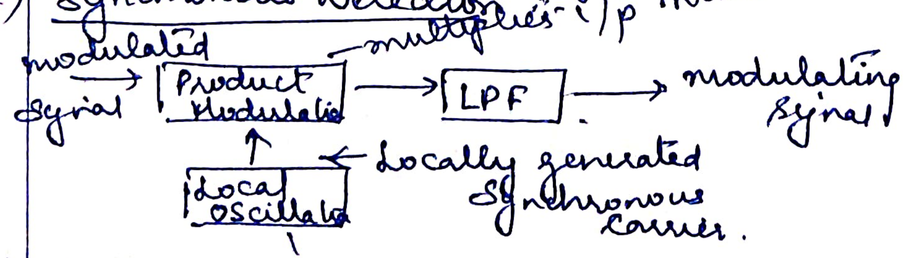
2) Envelope Detector:





$$\frac{1}{f_c} << R_C << \frac{1}{W}$$

8) **Synchronous Detection:** multiplies i/p modulated s/g.



9) **Virtues & limitations of Amplitude modulation:**

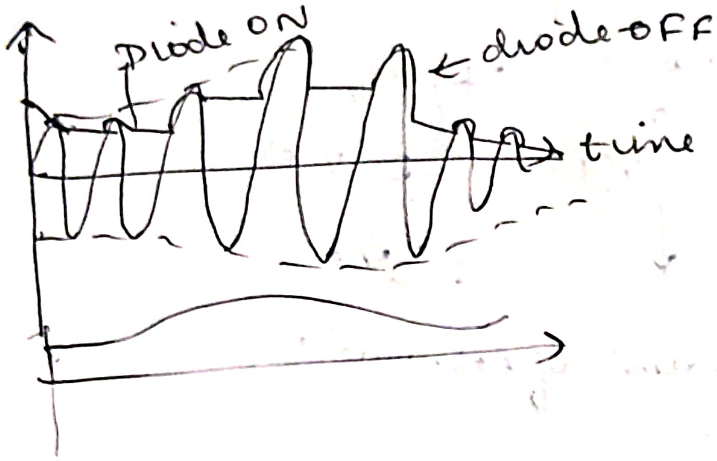
2 limitations

1) AM is wasteful of power 2) wasteful of BW

$$\therefore V_2(t) = a E_c[1+k_a x(t)]\cos(2\pi f_c t) + \frac{b E_c^2}{2}\left[1+2k_a x(t) + k_a^2 x^2(t)\right]\left[1+\cos(4\pi f_c t)\right]$$

Envelope detector :-

* +ve cycle fwd biased
* Capacitor charge → diode stops.



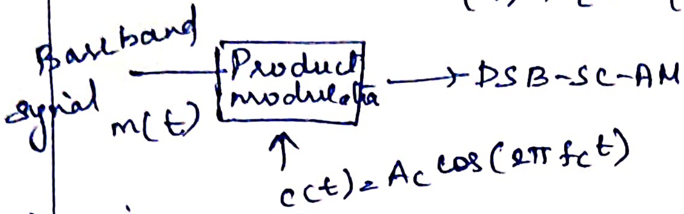
Charging time constant,

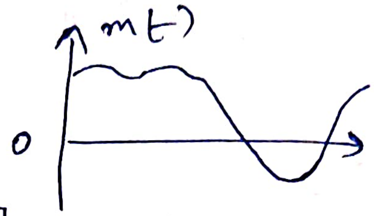
$$R_s C \ll \frac{1}{f_c}$$

IV) Double Sideband Suppressed Carrier (DSB-SC-AM) 9

$$s(t) = m(t) \cdot c(t)$$
$$= m(t) \cdot A_c \cos(2\pi f_c t)$$

$\left[\begin{array}{l} m(t) \to \text{msg signal} \\ c(t) \to \text{carrier wave} \end{array}\right]$

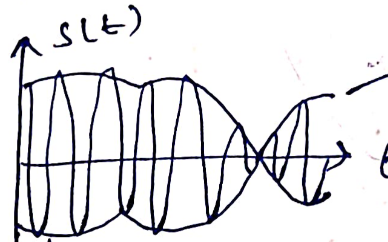




Tx Power & BW

Fourier T/f

$$S(f) = \frac{1}{2} A_c [M(f - f_c) + M(f + f_c)]$$



$$\boxed{\therefore S(f) = \frac{E_c E_m}{2} [\cos 2\pi (f_c - f_m) + \cos 2\pi (f_c + f_m) t]}$$

$$s(t) = c(t) \cdot m(t).$$
$$= E_c \cos(2\pi f_c t) \cdot E_m \cos(2\pi f_m t)$$

Power Calculation :- $P_t = P_c \left[1 + \frac{m_a^2}{2}\right]$

$$P_t' = \frac{m_a^2}{2} \left[\frac{E_c^2}{2}\right]$$

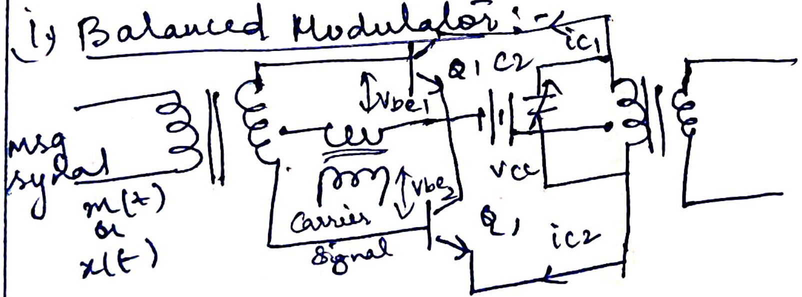
$$\boxed{P_t' = P_c \cdot \frac{m_a^2}{2}}$$

Power saving :- $\frac{P_t - P_t'}{P_t} = \frac{2}{2 + m_a^2}.$

Power saving ↑sed to 33.3% to 66.7%.

1) Generation of DSB-SC-AM

i) Balanced Modulator :-



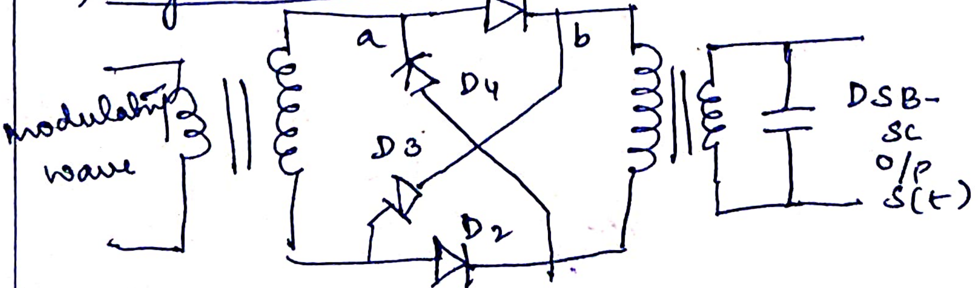
$$V_0 = 2a\, k x(t) + 4 b k x(t)$$

Modulation Signal

$E_c \cos(2\pi f_c t)$

DSB-SC signal.

2) Ring Modulator :-



Power calculation :-

$$P_t' = P_{LSB} + P_{USB}$$

$$= \frac{m_a^2 E_c^2}{8} + \frac{m_a^2 E_c^2}{8}$$

$$P_t' = \frac{m_a^2 E_c^2}{4}$$

$$\boxed{P_t' = P_c \cdot \frac{m_a^2}{2}.}$$

Power saving :-

$$\frac{P_t - P_t'}{P_t} = \frac{\left(1+\frac{m_a^2}{2}\right)P_c - \frac{m_a^2}{2}P_c}{\left(1+\frac{m_a^2}{2}\right)P_c}$$

$$= \frac{P_c}{\left(1+\frac{m_a^2}{2}\right)P_c} = \frac{1}{1+\frac{m_a^2}{2}}$$

$$= \frac{2}{2+m_a^2}.$$

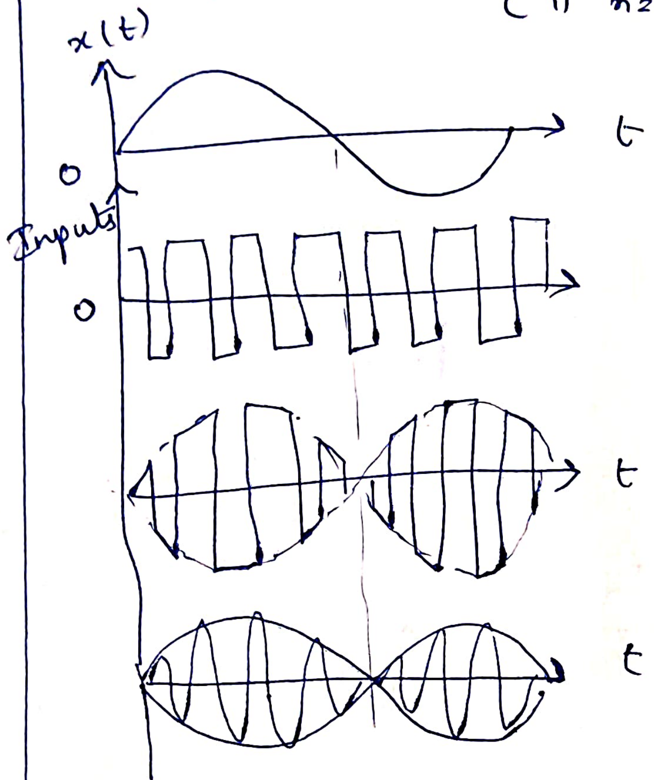
Power saving = 66.7 %.

Square wave carrier signal.

$$c(t) = \frac{4}{\pi} \sum_{n\geq 1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left[2\pi f_c(t)(2n-1)\right]$$

$$\text{O/p } V_0 = s(t) = x(t) \cdot c(t)$$

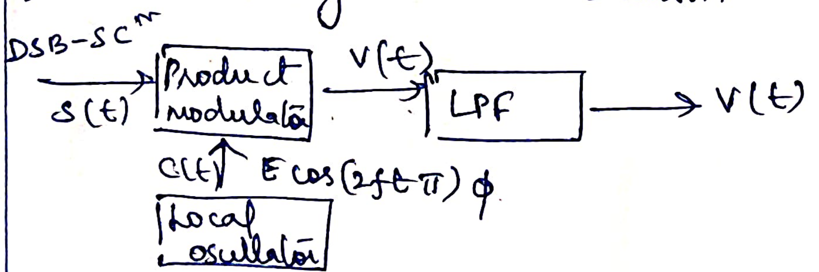
$$= x(t)\left\{\frac{4}{\pi} \sum_{n\geq 1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left[2\pi f_c(t)(2n-1)\right]\right\}$$



$f_c \to \phi$

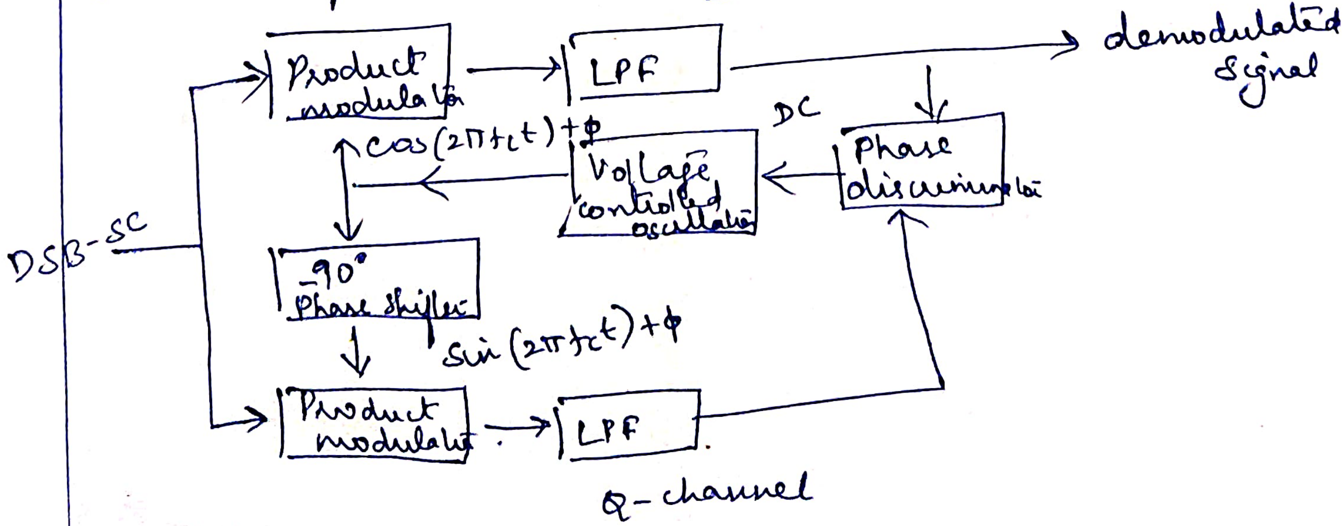
2) Detection of DSB SC-AM waves.

1) Coherent or synchronous detection.



$$V_0(t) = \frac{1}{2} x(t) E_c E_c' \cos\phi.$$

2) Costas Loop Detection.



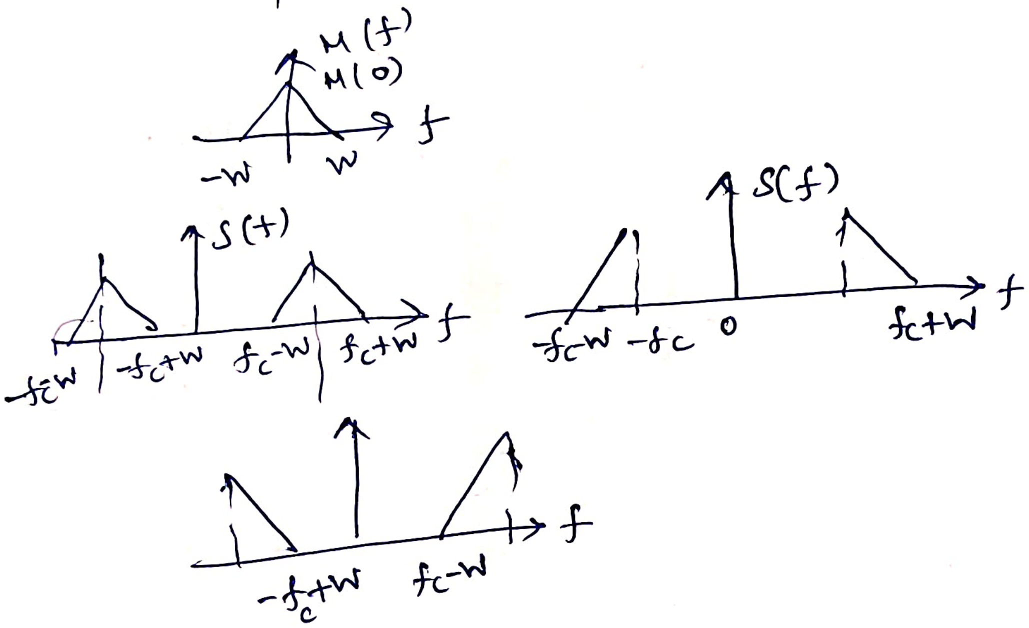
Single Side Band (SSB) system :-

* One sideband & Transmit the other.

Power calculation :

$$P_t = P_c\left[1+\frac{m_a^2}{2}\right]$$

$$P_t'' = \frac{m_a^2}{4}P_c \; , \; P_{USB} \; , \; P_{LSB}$$



Power saving :-

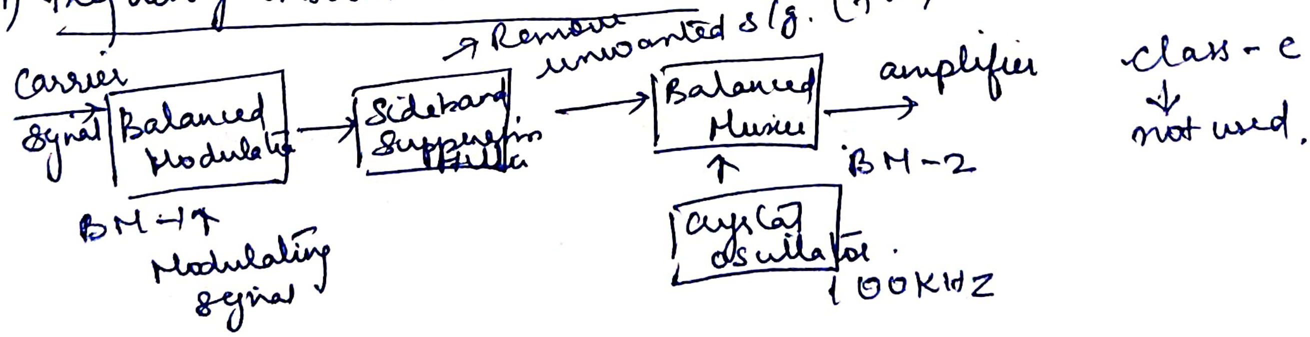
$$= \frac{P_t - P_t''}{P_t} = \frac{\left(1+\frac{m_a^2}{2}\right)P_c - \frac{m_a^2}{4}P_c}{\left(1+\frac{m_a^2}{2}\right)P_c}$$

$$= \frac{4+m_a^2}{4+2m_a^2}$$

$$= 83.3\%$$

1) Generation of SSB - SC - AM.

i) Frequency discrimination method : (Filter Method)



2) Phase Discrimination :- (or) phase shift :-

Balanced modulator :-

* eliminates unwanted harmonics,

$$V_{be_1} = x(t) + c(t) \Rightarrow x(t) + E_c \cos(2\pi f_c t) \quad -(1)$$

I/p at Transistor at Q_2

$$V_{be_2} = -x(t) + c(t)$$
$$= -x(t) + E_c \cos(2\pi f_c t) \quad -(2)$$

Collector current,

$$i_{C_1} = a V_{be_1} + b V_{be_1}^2 \quad -(3)$$
$$i_{C_2} = a V_{be_2} + b V_{be_2}^2 \quad -(4)$$

sub eq (1) & (2) in eqn (3) & (4).

$$i_{C_1} = a x(t) + a E_c \cos(2\pi f_c t) + b x^2(t) + b\, 2 x(t) E_c \cos(2\pi f_c t) + b E_c 2 \cos 2(2\pi f_c t)$$

IIIly

$$i_{C_2} = -a x(t) + a E_c \cos(2\pi f_c t) + b x^2(t) - b 2 x(t) E_c \cos(2\pi f_c t) + b E_c 2 \cos 2(2\pi f_c t)$$

O/p voltage,

$$V_o = K(i_{C_1} - i_{C_2})$$

$$\boxed{\therefore V_o = 2aK x(t) + 4bK x(t) E_c \cos(2\pi f_c t)}$$

Ring Modulator :- (Chopper modulator)

+ve half cycle :-

i) D_1 & D_2 → fwd biased → o/p current.

ii) D_3 & D_4 → Reverse biased does not conduct.

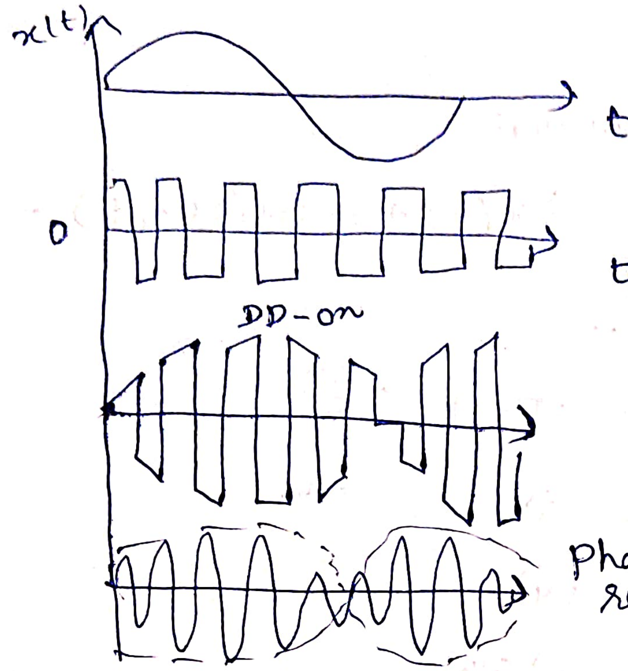
-ve half cycle :-

i) D_1 & D_2 → reverse biased → do not conduct.

D_3 & D_4 → Fwd biased → Conduct.

Square wave carrier signal,

$$c(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_c (t)(2n-1)]$$



2) Detection of DSB-SC-AM wave

1. Coherent or synchronous detection

2. Cost as loop detection.

$V(t) = S(t).C(t)$

$V(t) = x(t).E_c \cos(2\pi f_c t) E_c' \cos(2\pi f_c t + \phi)$

$\because \cos A.\cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$\therefore V(t) = \frac{1}{2} x(t).E_c.E_c' [\cos(4\pi f_c t + \phi) + \cos\phi]$

$V(t) = \frac{1}{2} x(t).E_c.E_c' \cos(4\pi f_c t + \phi) + \frac{1}{2} x(t).E_c E_c'.\cos\phi.$

$$\boxed{\text{Filter o/p} = \frac{1}{2} x(t).E_c E_c' \cos\phi}$$

ii, Costas loop detection:-

One coherent detector → I-channel — L.O.C → channel is zero

Second " → Q-channel. → L.C → channel is not zero.

Case i, I channel o/p $= S(t).\cos[2\pi f_c t + \phi]$

$= E_c \cos(2\pi f_c t).x(t).\cos[2\pi f_c t + \phi]$

Low pass filter, $\cos\phi = \frac{E_c x(t)}{2}$.

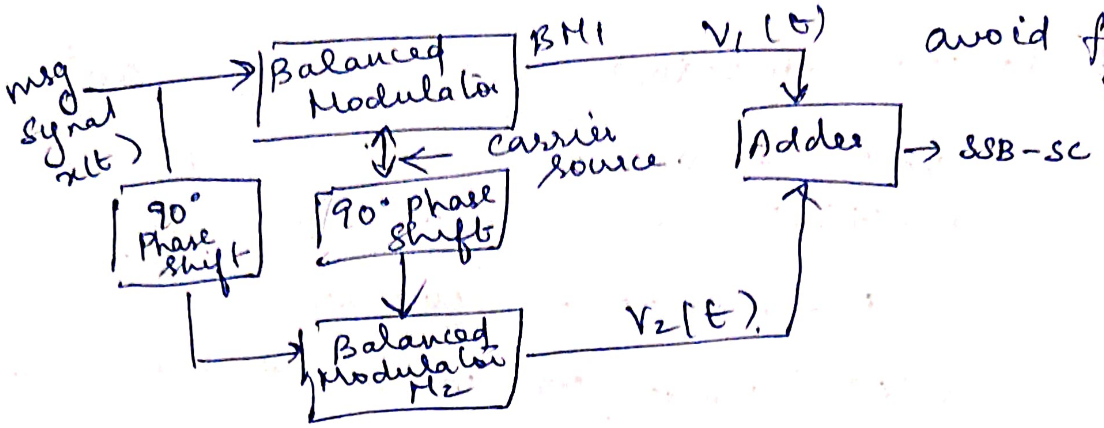
Q-channel o/p:- $= S(t).\sin[2\pi f_c t + \phi]$

$= E_c \cos(2\pi f_c t).x(t).\sin(2\pi f_c + \phi)$

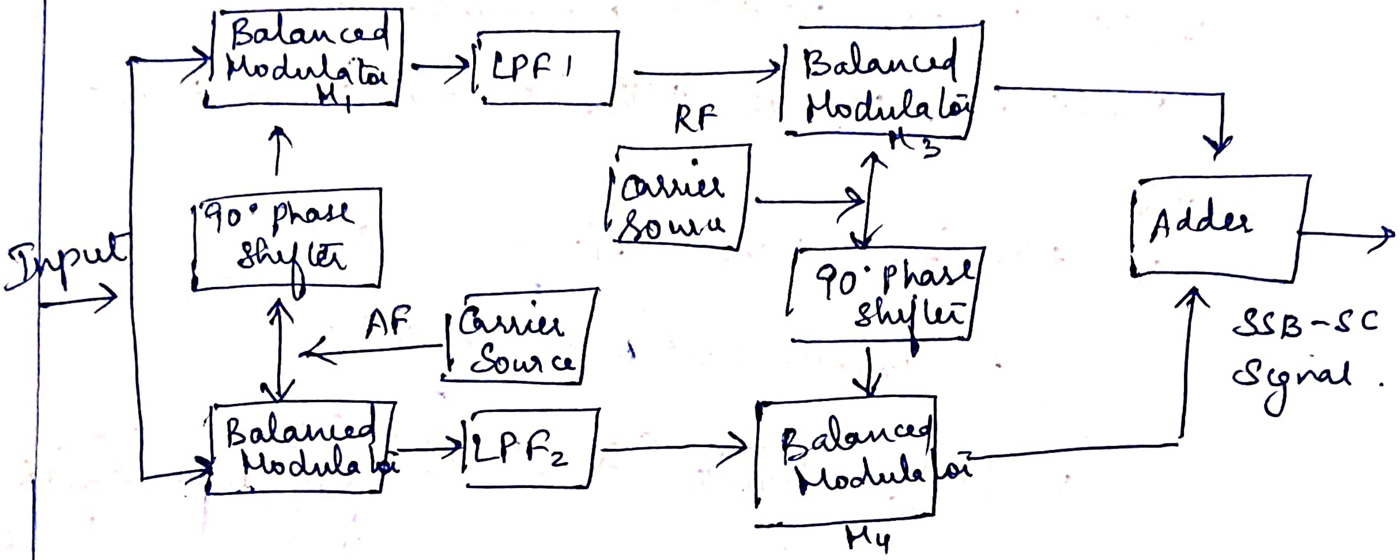
$= \frac{E_c x(t)}{2}[\sin(4\pi f_c t + \phi) + \sin\phi]$

Case ii, o/p $= \frac{E_c x(t)\phi}{2}$

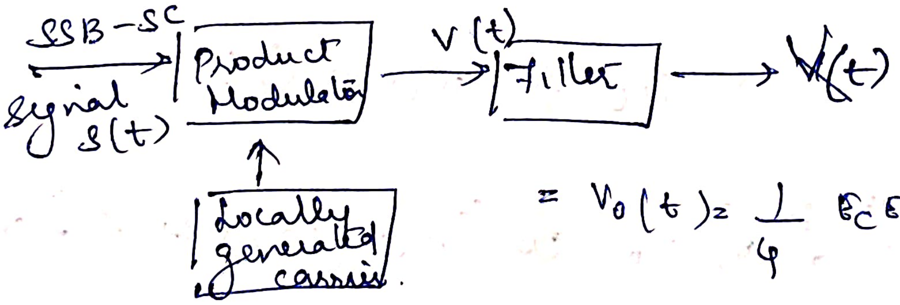
avoid filter Method.



3) Modified Phase Shift Mtd (or) weaver's Mtd.



2) Detection of SSB-SC-AM wave:-

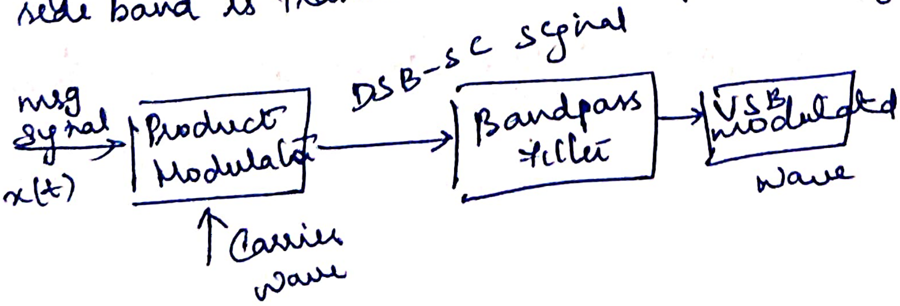


$= V_0(t) = \frac{1}{4} E_c E_c' x(t)$

$$V(t) = \frac{1}{4} E_c E_c' \left[x(t) \cos 4\pi f_c(t) + x(t) \pm x_h(t) \sin 4\pi f_c(t) \right]$$

V) Vestigial sideband modulation :-

one of the sideband is partially suppressed & a vestige of the other side band is transmitted to compensate for the suppression.



Balanced modulator 1, 90°

$V_c(t) = V_c \cos \omega_c t$ — (1)

$V_m(t) = V_m \cos \omega_m t$ — (2)

$V_1(t) = V_c \cos \omega_c t \cdot V_m \cos \omega_m t$.

$V_1(t) = \frac{1}{2} V_m V_c [\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t$ — (3)

BM 2:

$V_c(t) = V_c \cos\left(\omega_c + \overset{90^\circ}{\pi/2}\right)t = -V_c \sin \omega_c t$ — (4)

$V_m(t) = V_m \cos(\omega_m + \pi/2)t = -V_m \sin \omega_m t$ — (5)

O/P,

$V_2(t) = (-V_m \sin \omega_m t) \cdot (-V_c \sin \omega_c t)$

$= V_m \sin \omega_m t \cdot V_c \sin \omega_c t.$

$V_2(t) = \frac{1}{2} V_m V_c [\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t.$ — (6)

(3) & (6).

$V_m V_c \cos(\omega_c - \omega_m)t.$

Weaver's Mtd :-

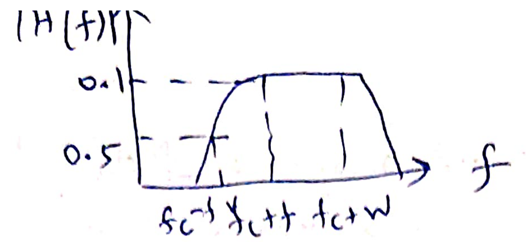
O/p of BM1 → $V_m V_o (\cos((\omega_o t - \omega_m t) + 90^\circ) - \cos((\omega_o t + \omega_m t) + 90^\circ]$

BM2 = $V_m V_o [\cos[(\omega_o t - \omega_m t) - \cos(\omega_o t + \omega_m t)]]$

BM3 = $\sin[(\omega_c + \omega_o - \omega_m)t + 90^\circ] + \sin[(\omega_c - \omega_o + \omega_m)t - 90^\circ$

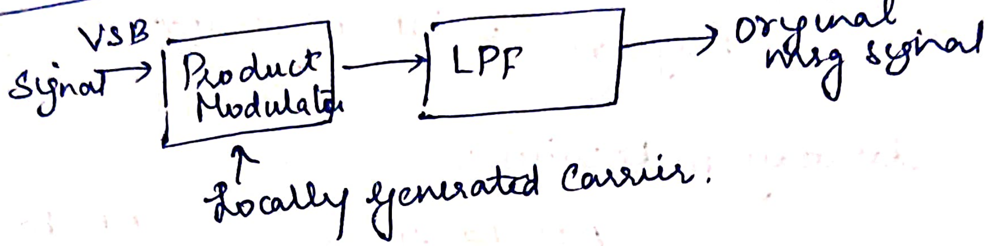
BM4 = $\sin[\omega_c + \omega_o - \omega_m)t + 90^\circ] + \sin[(\omega_c - \omega_o + \omega_m)t + 90^\circ$

$\therefore V_o(t) = 2\cos(\omega_c + \omega_o - \omega_m)t.$



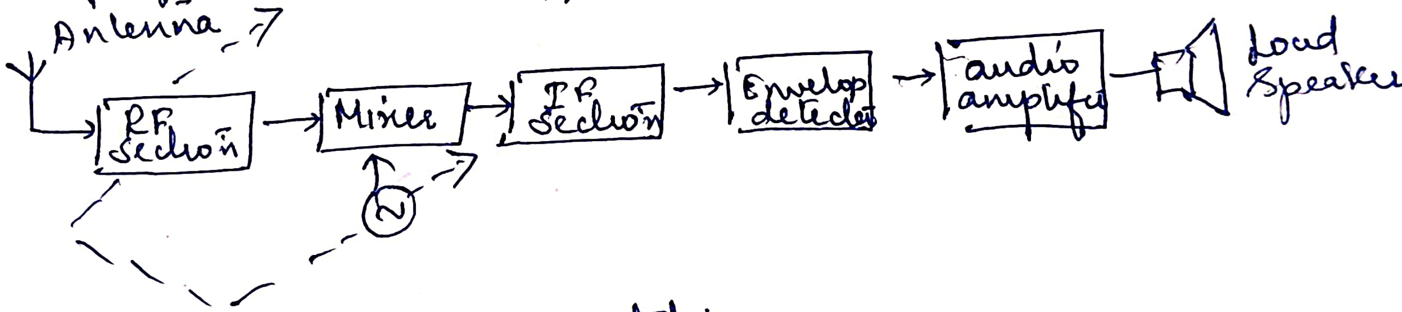
modulated wave: $S(t) = \frac{1}{2} E_c x(t) \cos(2\pi f_c t) \pm \frac{1}{2} E_c x'(t) \sin(2\pi f_c t)$

Demodulated:

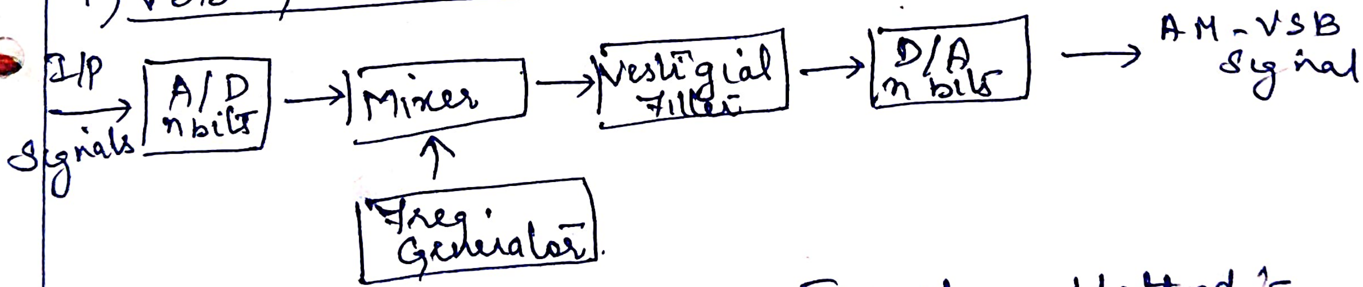


VII **Superhetorodyne Receiver:-**

1. Carrier Freq. tuning → desired signal
2. Filtering → desird signal from modulated signal.
3. Amplification → loss of signal power during Txm.



1) **VSB Generation - Filter mtd :-**



2) **VSB Generation - Hilbert Transform Method :-**

* To implement modulator - Hilbert Transform is needed to determine the Transfer fn, only the real part (or) imaginary one is expressed,

$$F(w) = R(w) + jW(w).$$

$$\therefore X(\omega) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{R(y)}{w-y} dy.$$

VSB :-

2 condition.

i) Sum of values of magnitude Response $|H(f)|$ @ two frequencies equally displaced above or below unity.

ii) Phase response arg $(H(f))$ is linear.

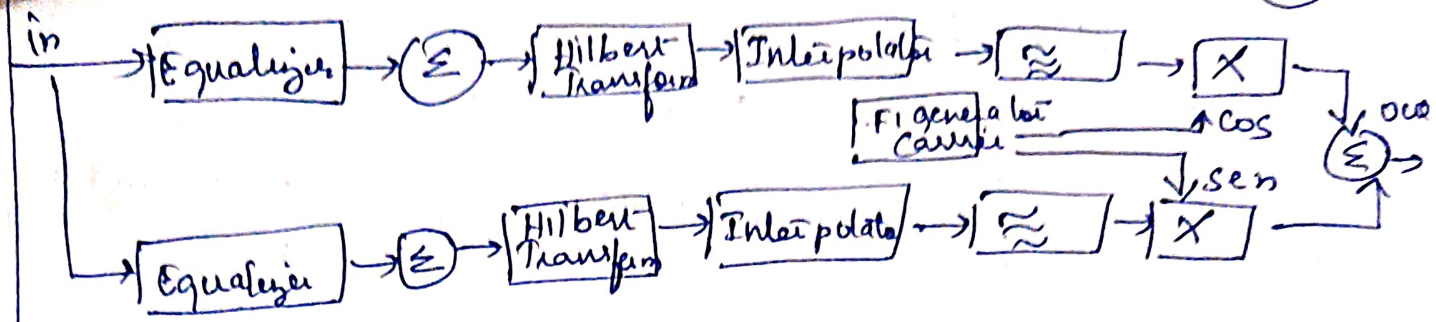
$$B_T = W + f$$

Msg BW — width of vestigial side band.

Superhetorodyne Receiver :-

	AM Radio	FM Radio
RF Carrier range	0.525-1.605 MHz	88 - 108 MHz
Mid band freq.	0.455 MHz	10.7 MHz
IF Bandwidth	10 KHz	200 KHz

Modulator Block diagram :-



VIII **Angle Modulation :- (Representation of FM & PM signals, Spectral characteristics of angle modulated signals).**

α Carrier is Varied in accordance to instantaneous Value of msg (or) base band signal.

- Frequency Modulation (FM)
- Phase modulation (PM)

Adv:

Noise Reduction

Improved system

Disadv:

↑sed BW

more complex circuit

Applications

Radio broad casting

TV sound Txm.

1) Phase Modulation :-

$$K_p = \frac{\Delta\theta}{\Delta E} \text{ (or) } \frac{\Delta\theta}{\Delta V}$$

2) Frequency modulation :-

$$f_i(t) = f_c + K_f x(t)$$

$$S(t) = E_c \cos\left[2\pi f_c t + K\pi K_f \int_0^t x(\tau).d\tau\right]$$

Freq. deviation :-

$$S(t) = E_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\, t\right]$$

2) Narrow Band Frequency modulation :-

FM Signal : $S(t) = E_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right]$

Hillbert Transform :-

$f(t)=0$ for $t<0$. → Hilbert Transform,

$R(\omega)$ → even fn, $X(\omega)$ → odd one.

1) $R(\omega) = 2\int_0^\infty f_e(t).\cos\omega t.dt$; $f_e(t) = \frac{1}{\pi}\int_0^\infty R(\omega)\cos\omega t.d\omega$

2) $X(\omega) = -2\int_0^\infty f_o(t)\sin\omega t.dt$ $f_o(t) = \frac{1}{\pi}\int_0^\infty X(\omega)\sin\omega t.d\omega$

Casual fn, $f(t)=0$, $t>0$.

$$\boxed{f(t) = 2.f_e(t) . 2.f_o(t)}$$

$$R(\omega) = \int_{-\infty}^{\infty} e^{j\omega t} f_o(t).dt = \int_{-\infty}^{\infty} e^{j\omega t} f_o(t).\sin(t).dt$$

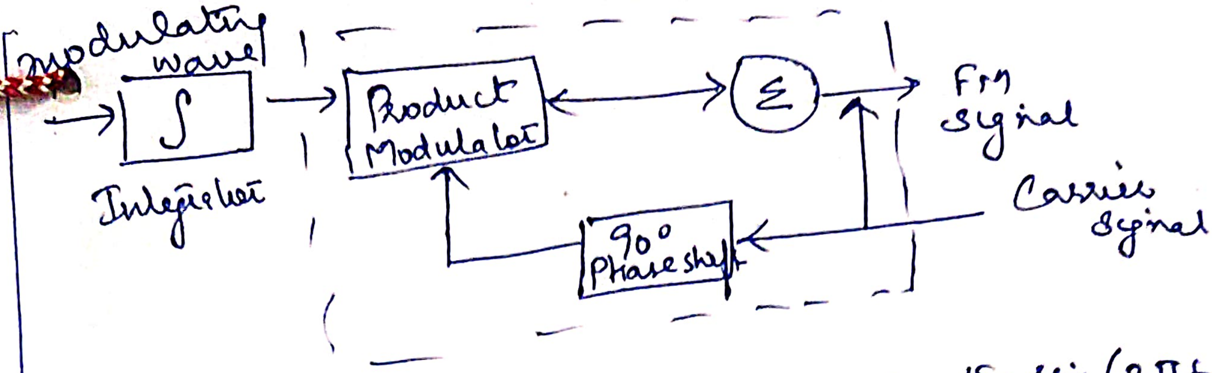
$$R(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} jX(y)\left[\frac{2}{j(\omega-y)}\right]dy$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{X(y)}{\omega-y}.dy.$$

$$\therefore X(\omega) = \frac{-1}{\pi}\int_{-\infty}^{\infty}\frac{R(y)}{\omega-y}.dy.$$

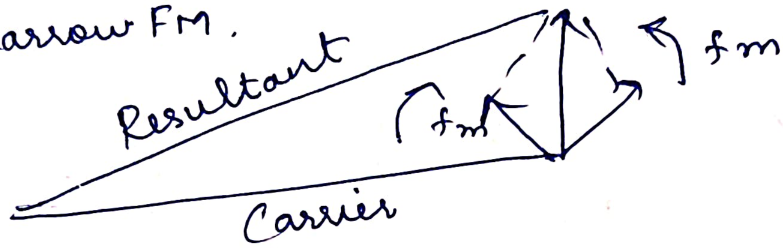
Spectral characteristics :-

* Single freq. modulated S/g produces an infinite no. of pairs of side frequencies & infinite b/w.



$$S(t) = E_c \cos(2\pi f_c t)\cos(\beta \sin(2\pi f_m t)) - E_c \sin(2\pi f_c t)\sin(\beta \sin(2\pi f_m t))$$

i) Narrow FM.



ii) AM wave:

3) Wide Band Frequency modulation :-

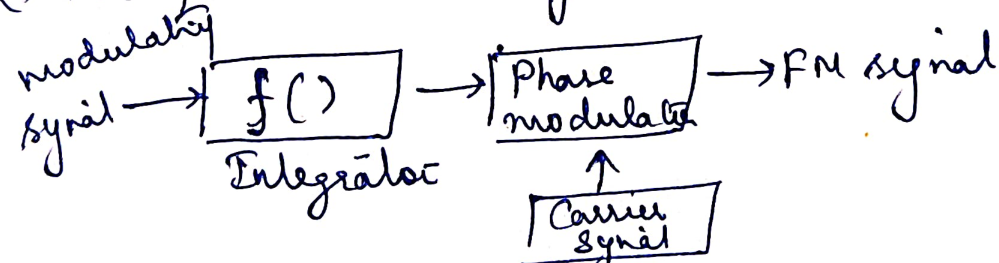
Sinusoidal Modulating signal

$$S(t) = E_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$$

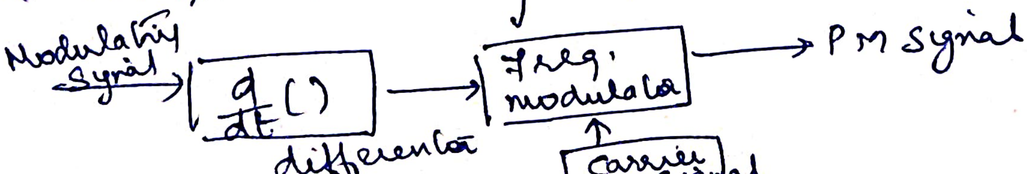
$$C_n = f_m E_c \int_{-\frac{1}{2} f_m}^{\frac{1}{2} f_m} \exp[j\beta \sin(2\pi f_m t) - j 2\pi n f_m t] \cdot dt.$$

Relationship b/w PM & FM :-

i) FM Generation using Phase modulator:



ii) PM Generation using Freq. Modulator:



Pre-envelop, Complex envelope Representation of AM Techniques.

Let $x(t)$, is

$$x(t) = x(t) + j\,\hat{x}(t)$$

Natural - envelope:

$$\tilde{x}(t) = m(t).e^{j\phi} = m(t)\left[\cos\phi + j\sin\phi\right]$$

$$= m(t)\cos\phi + j.m(t).\sin\phi.$$

complex envelope:

$$x(t) = x_+(t).e^{-j2\pi f_c t}$$

$$= m(t).e^{+j(2\pi f_c t + \phi(t))}.e^{-j2\pi f_c t}$$

$$= m(t).e^{j\phi(t)}.$$

Random Process & Sampling.

UNIT - II

I Review of Probability & Random Process :-

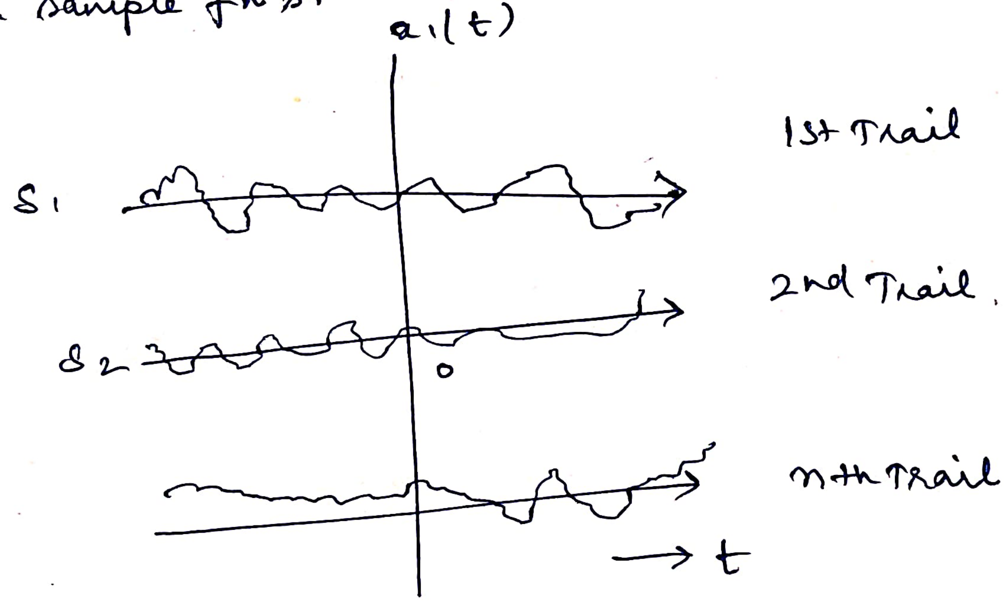
Probability :-

The study of uncertainty that provides Mathematical models for random phenomena & experiments.

$$P(A) = \frac{\text{no. of possible outcomes}}{\text{Total no. of outcomes}}.$$

Random Process :-

RP $X(t)$ as an ensemble of time functions together with probability rule that assigns a probability to any meaningful event associated with observation of one of the sample fn's.

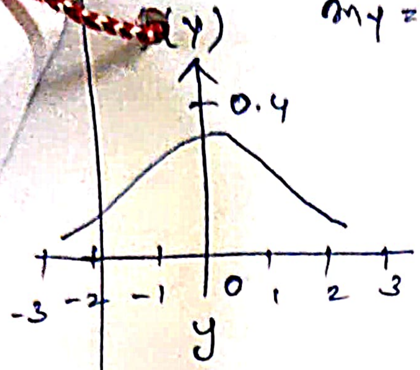


II Gaussian & White Noise characteristics :-

$x(t)$ for interval $t=0$ to $t=T$.

$$Y = \int_0^{T} g(t) \cdot x(t) \cdot dt.$$

Gaussian distribution $\Rightarrow$ $f_Y(y) = \frac{1}{\sqrt{2\pi}\sigma_Y} \exp\left[-\frac{(y-m_Y)^2}{2\sigma_Y^2}\right]$



m_Y = mean, σ_Y^2 = Variance of random Variable.

$$f_Y(y) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{y^2}{2}\right)$$

Adv:- 1) Analytic Results possible
2) RP produced physical phenomena approximate.
3) Physical phenomena study of CS.

Properties:- P1: $X(t)$ is stable linear filter

P2: Random Variables $X(t_1), X(t_2) \ldots X(t_n)$.

$$m_x(t_i) = E[X(t_i)], \quad i = 1, 2 \ldots n.$$

cross-covariance function is,

$$= E[X(t_i) - m_X(t_i)(X(u_k) - m_X(u_k)]$$

$$= R_{XY}(t_i, u_k) - m_X(t_i)\, m_Y(u_k).$$

P3: wide-sense stationary

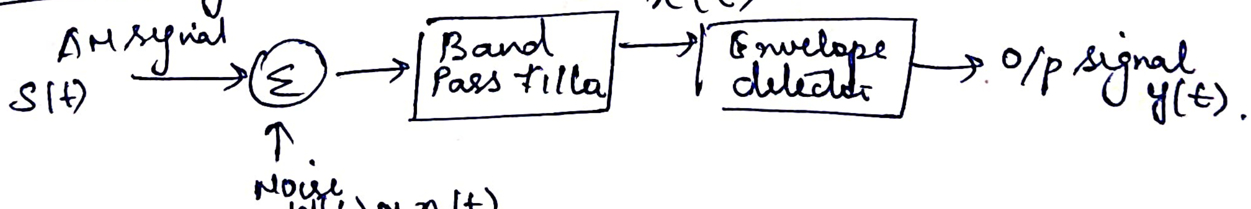
P4: $E[X(t_k) - m_X(t_k))(X(t_i) - m_X(t_i)] = 0, \quad i \neq k.$

iii) Noise in amplitude modulation systems:-

Full AM signal, both sidebands & Carrier wave transmitted as,

$$S(t) = E_c[1 + \underbrace{K_a}_{\text{Constant}} \underbrace{x(t)}_{\text{msg signal}}] \cos(2\pi f_c t).$$

Noise analysis:-



Avg power of AM signal $S(t) = \frac{E_c^2}{2}$

avg power of information, $E_c K_a x(t) \cdot \cos(2\pi f_c t)$ is,

$$\frac{E_c^2 K_a^2 P}{2}$$

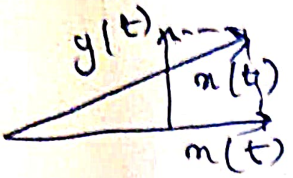
Avg power of AM Signal $S(t)$, $\frac{E_c^2(1 + K_a^2 P)}{2}$.

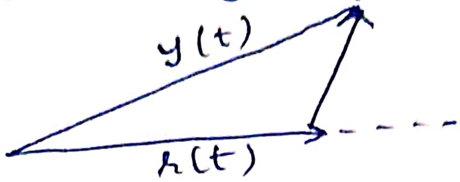
Channel Signal-to-noise AM, $\frac{E_c^2(1 + K_a^2 P)}{2 W N_0}$

Filtered signal is

$x'(t) = s(t) + n(t)$

$= [E_c + E_c K_a x(t) + n_I(t)] \cos(2\pi f_c t) - n_Q(t) \cdot \sin(2\pi f_c t)$.





O/p signal to noise ratio,

$$(SNR)_{O,AM} = \frac{E_c^2 K_a^2 P}{2WN_0}$$

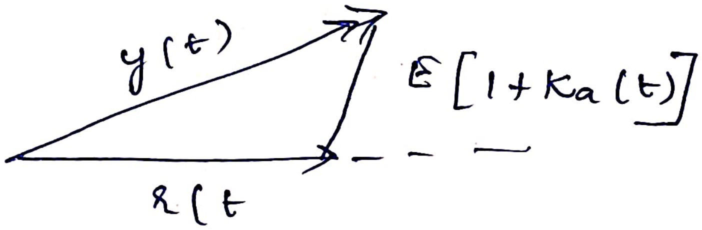
<u>Figure of Merit</u>:

$$\left.\frac{(SNR)_O}{(SNR)_C}\right|_{AM} = \frac{K_a^2 P}{1+K_a^2 P}.$$

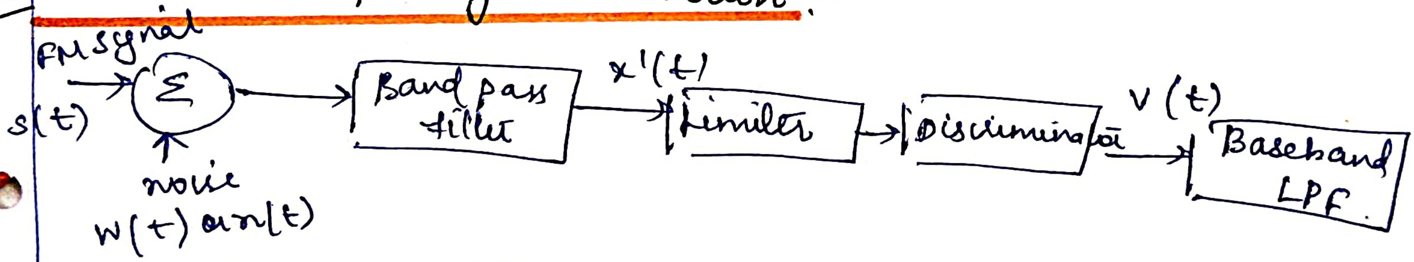
i) <u>Threshold effect</u>:-

* Loss of a msg in an envelope detector that operates at low carrier to noise ratio is refer as T.E.

$n(t) = r(t) \cos[2\pi f_c t + \psi(t)]$



IV <u>Noise in Frequency modulation</u>:-



<u>2 Components</u>:-

1) Slope N/W   2) Envelope detector.

The Filtered Noise $n(t) = n_I(t) \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t)$

envelope & phase as, $n(t) = r(t) \cdot \cos[2\pi f_c t + \psi(t)]$

Phase $\psi(t) = \tan^{-1}\left[\frac{n_Q(t)}{n_I(t)}\right]$

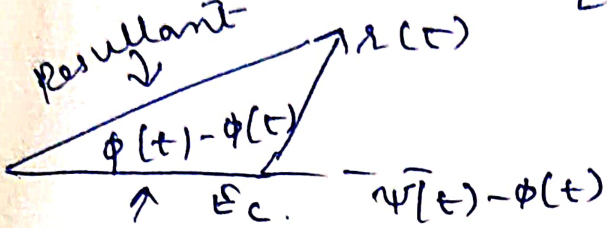
$r(t) \rightarrow$ rayleigh distributed, $\psi(t)$ is uniformly distributed.

$$\phi(t) = 2\pi k_f \int_0^t x(t)\cdot dt.$$

$$\therefore s(t) = E_c \cos\left[2\pi f_c t + \phi(t)\right]$$

Phasor representing $x'(t)$

$$\theta(t) = \phi(t) + \tan^{-1}\left\{\frac{r(t)\sin[\psi(t)-\phi(t)]}{E_c + r(t)\cos[\psi(t)-\phi(t)]}\right\}$$



$$\theta(t) = \phi(t) + \frac{r(t)}{E_c}\sin[\psi(t)-\phi(t)]$$

$$\therefore \theta(t) = 2\pi f_c t \int_0^t x(t)\,dt + \frac{r(t)}{E_c}\sin(\psi(t) - \phi(t)]$$

Sub $\phi(t)$ in above.

$$\theta(t) = 2\pi f_c t \int_0^t x(t)\,dt + \frac{r(t)}{E_c}\sin[\psi(t) - \phi(t)]$$

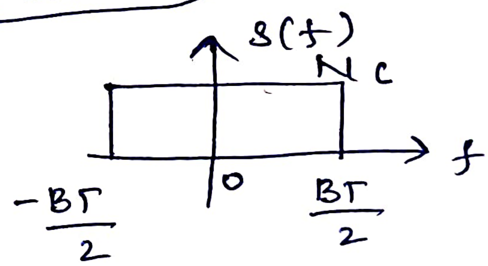
$$\therefore \underset{\text{noiseterm}}{n_d(t)} = \frac{1}{2\pi E_c}\cdot\frac{d}{dt}\left\{r(t)\cdot\sin[\psi(t)-\phi(t)]\right\}$$

$n_Q(t)$ is,

$$n_Q(t) = r(t)\cdot\sin[\psi(t)]$$

$$\boxed{\therefore n_d(t) = \frac{1}{2\pi E_c}\cdot\frac{d}{dt}n_Q(t)}$$

$$S_{Nd}(f) = \frac{f^2}{E_c^2}S_{NQ}(f).$$



Power Spectral density $\rightarrow$ $S_{nd}(f) = \begin{cases} \frac{N_0 f^2}{E_c^2}, & |f| \leq \frac{B_T}{2} \\ 0, & \text{otherwise}. \end{cases}$

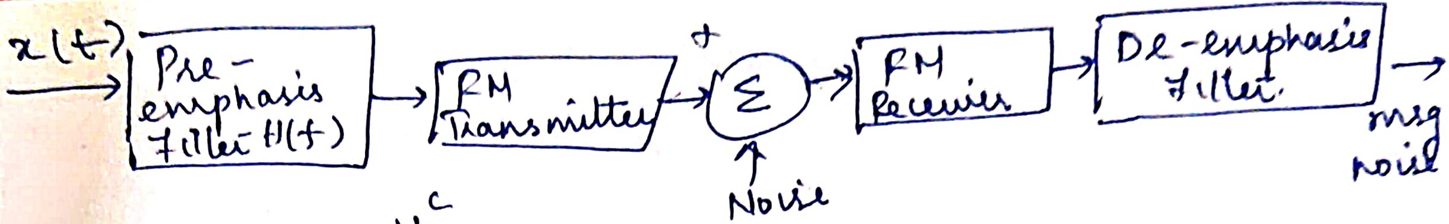
$$\text{Avg Power} = \frac{N_0}{E_c^2}\int_{-W}^{W} f^2\cdot df.$$

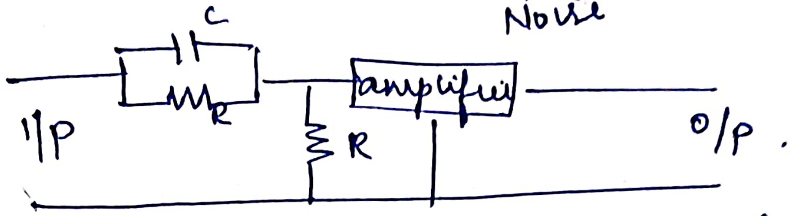
$$\text{Fig of Merit} = \left.\frac{(SNR)_o}{(SNR)_c}\right|_{FM} = \frac{\frac{3E_c^2 k_f^2 P}{2N_0 W^3}}{\frac{E_c^2}{2WN_0}} = \frac{3k_f^2 P}{W^2}$$

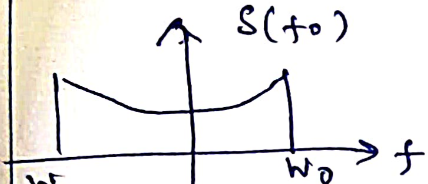
1) Capture effect :-

2) FM threshold effect :-

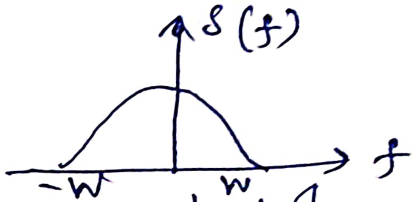
v) Pre-emphasis & De-emphasis :-







FM receiver o/p



Power spectral of msg signal

$$S_{nd}(f) = \begin{cases} \frac{N_o f^2}{E_c{}^2}, & |f| \leq \frac{B_T}{2} \\ 0, & \text{otherwise.} \end{cases}$$

O/p power noise

$$\boxed{\therefore I = \frac{2W^3}{3\int_{-W}^{W} f^2 |H_{de}(f)|^2 \, df}}$$

vi) Threshold effect in angle modulation:-

When i/p signal-to-noise ratio (SNR) falls below certain value.

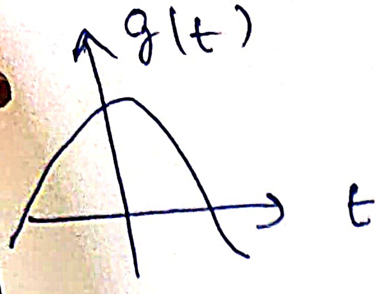
vii) Low pass sampling :-

* Sampling process from analog to digital conversion.

1) Sampling Theorem :-

$g(nT_s)$, $n = 0, \pm 1, \pm 2 \ldots\ldots$

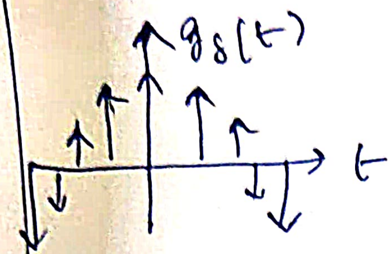
Sampling Process $\Big] \rightarrow$ $g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\, \delta(t - nT_s)$.



$$g(nT_s)\,\delta(t-nT_s) = g(t)\,\delta(t-nT_s)$$

Delta functions

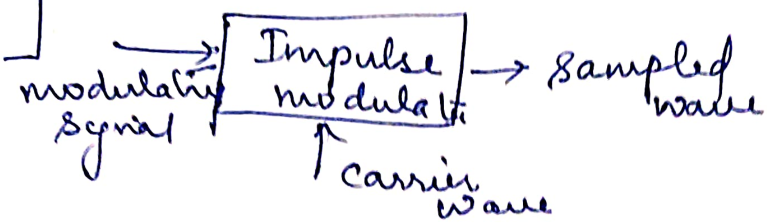
$$G_\delta(f) = f_s \sum_{n=-\infty}^{\infty} G(f - mf_s).$$



Fourier Transform,

$$F\left[\sum \delta(t-nT_s)\right] = \sum_{n=\infty}^{\infty} \exp(-j2\pi n f T_s)$$

Impulse modulator



$$\boxed{G(f) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-\frac{j n\pi}{W}\right), -W<f<W}$$

viii Aliasing (or) Foldover error.

* Sample $g(t)$ is not strictly bandlimited if the sampling frequency f_s is less than $2W$ then an error called aliasing (or) Foldover.
* The Phenomenon of a high freq. in the spectrum of original sample $g(t)$ taking on the identity of a lower frequency in spectrum of sampled signal $g(t)$ called aliasing.

Signal Reconstruction :-

$$F^{-1}[G(f)] = g(t) = \int_{-\infty}^{\infty} G(f)\exp(j2\pi f t)\,df$$

$$g(t) = \int_{-W}^{W} \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-j\frac{\pi n f}{W}\right) \exp(j2\pi f t)\,df.$$

$$= \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{\sin(2\pi W t - n\pi)}{2\pi W t - n\pi}$$

$$= \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \operatorname{sinc}(2Wt).$$

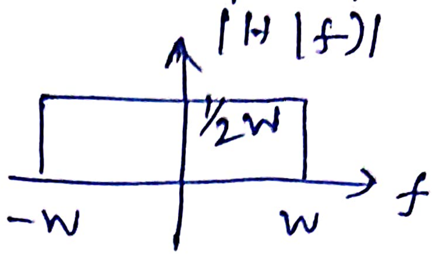
$$g(t) = \sum_{n=\infty}^{\infty} g\left(\frac{n}{2W}\right) \operatorname{sinc}(2Wt - u).$$

Interpolation fn.

Seq of Sample → [Ideal LPF BW] → Anglog filter

Reconstruction filter.

Ideal amp. response



Sampling Theorems :-

1. Band-limited Sample of finite energy, has no freq. Component higher than 'w' hertz, the values of the Sample at instant of time seperated by $\frac{1}{2}W$ sec.
2) no freq. higher the 'w' hertz.

Nyquist rate :- * Sampling rate 2w samples/sec, for signal b/w 'w' Hertz called Nyquist rate $\frac{1}{2W}$ called Nyquist interval.

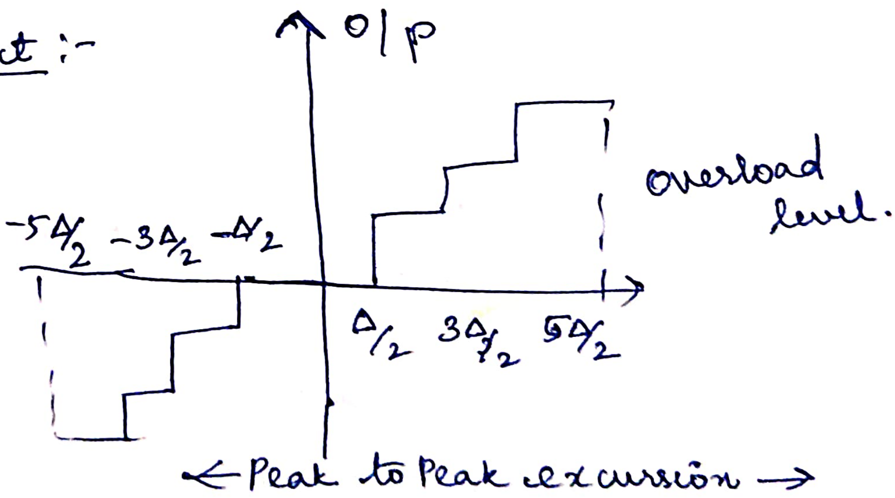
VIII Quantization :-

1) Uniform & Non-uniform Quantization :-

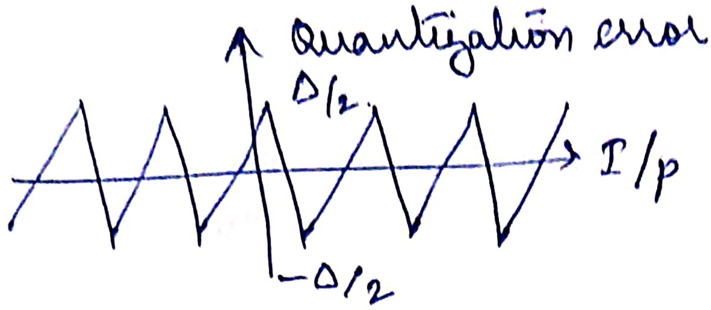
Sampled Signal → [Quantizer] → signal with finite level.

Conversion of analog sample into digital called Quantization.

Two-fold effect :-



The Separation b/w the decision threshold & separation b/w the representation level of Quantizer Called Step size.



Types of uniform Quantizer :-

1. Symmetric Quantizer of mid-Tread type.
2) Symmetric quantizer of mid-rise Type.

Quantization noise :-

The Transmitter is by rounding-off sample values of an analog base-band sample to nearest permissible.

$$e_q = m(t) - m_q(t).$$

$$-\frac{\Delta}{2} < e_q < \frac{\Delta}{2}.$$

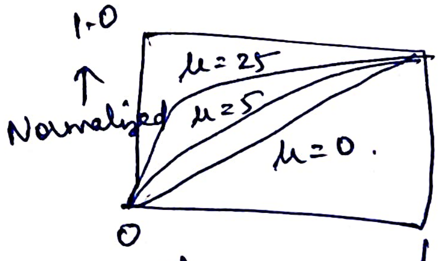
Mean square value error :-

$$\Delta = \frac{m_{max} - (-m_{max})}{L} = \frac{2m_{max}}{L}$$

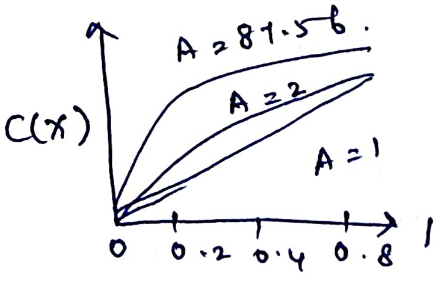
$$\text{Signal-to-noise Ratio} = \frac{P}{\sigma_Q^2}\left(\frac{3p}{m^2_{max}}\right) \times 2^{2R}.$$

Logarithmic Companding :-

1. μ-law Companding :-



2. A-law Companding :-



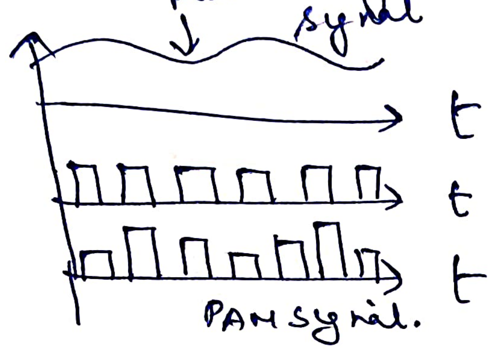
PAM, PPM, PWM, PCM - TDM, FDM.

~~PAM~~ (Pulse Analog modulation)

Pulse modulation

- Pulse Analog Modulation
  - → PAM
  - → PWM
  - → PPM
- Pulse Digital modulation
  - → PCM
  - → DPCM
  - → DM & ADM.

1) PAM :-



PAM signal :- $S(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\, h(t - nT_s)$

Flat-top PAM signal $S(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s) H(f)$.

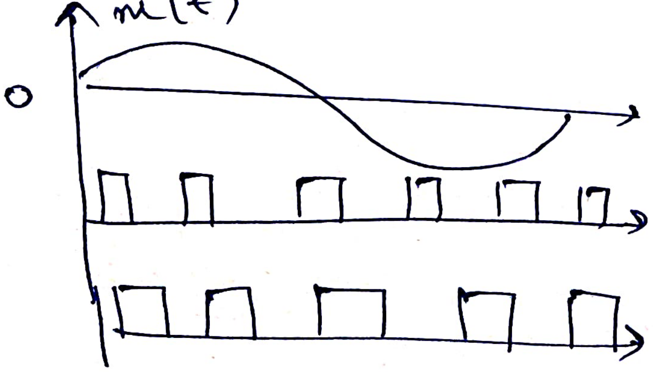
Naturally Sampled PAM signal :-

$$S(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \operatorname{sinc}(n f_s \tau)\, X(f - nf_s)$$

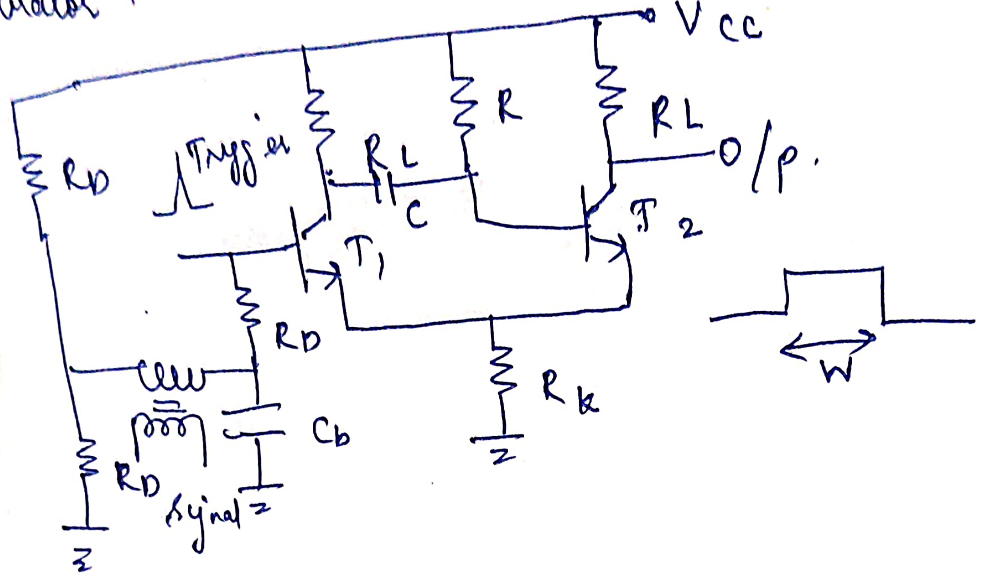
Demodulation of PAM :-

PAM signal $s(t)$ → [Reconstruction Filter] → [Equalizer] → msg signal (mt).

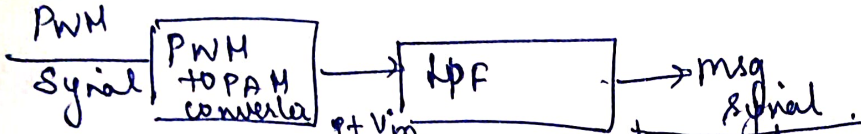
2) PWM :- (Pulse Duration modulation)

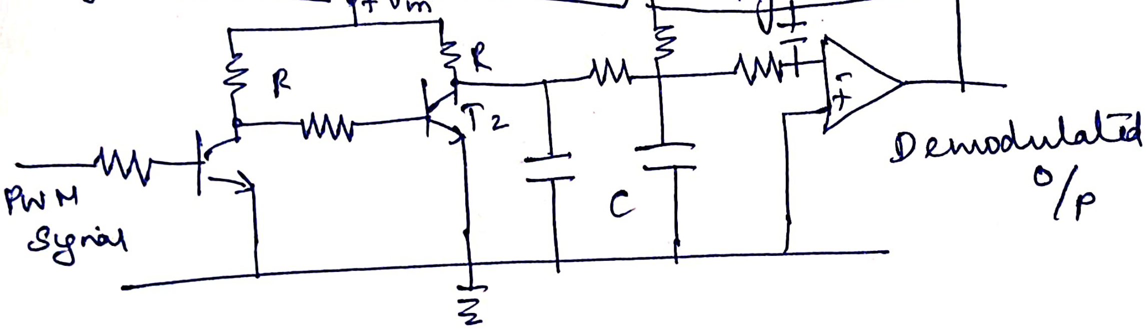


1) Direct Method of PWM generation using Monostable Multivibrator:

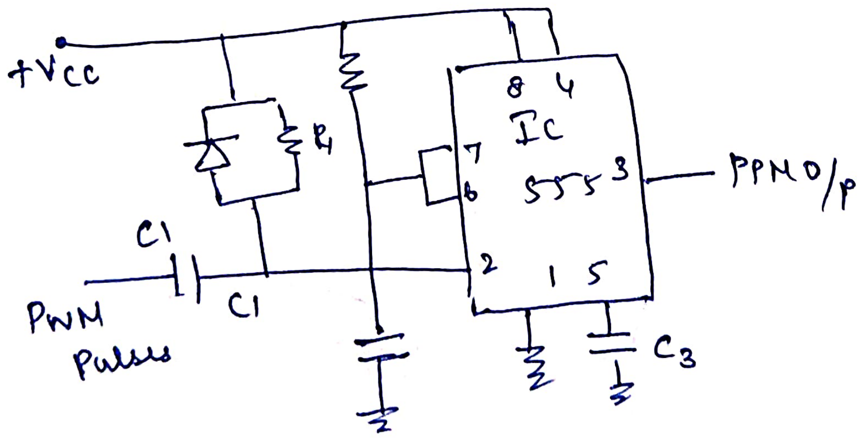


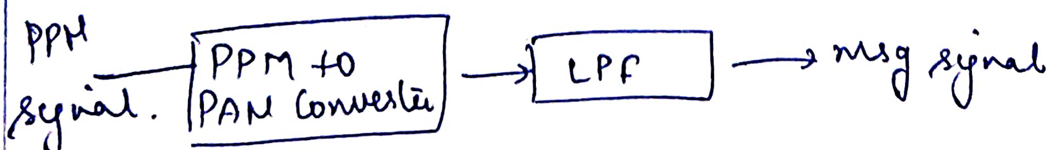
PWM Demodulation :-





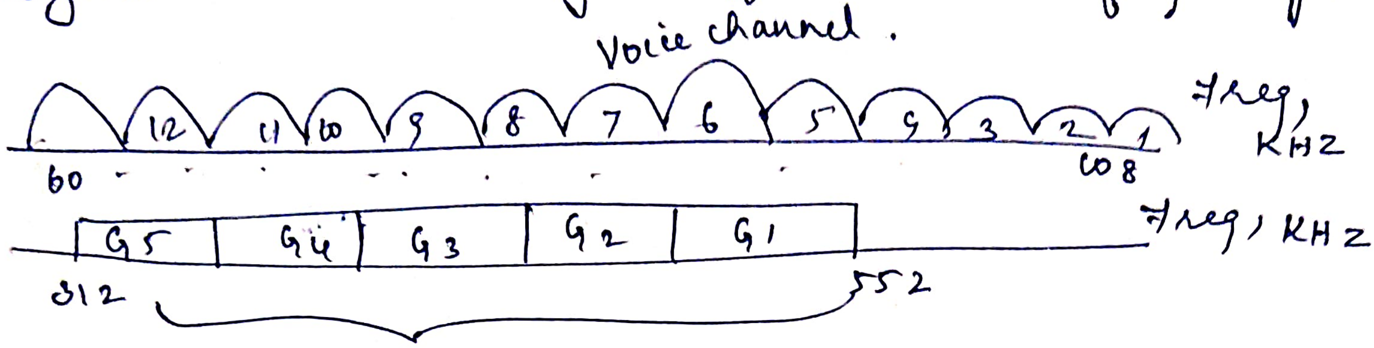
3) PPM (Pulseposition modulation):-





FDM (Frequency Devision Muliplexing) (FDM) 28

* Signal is modulated by a different Carrier frequency.



XII Nyquist criterion :-

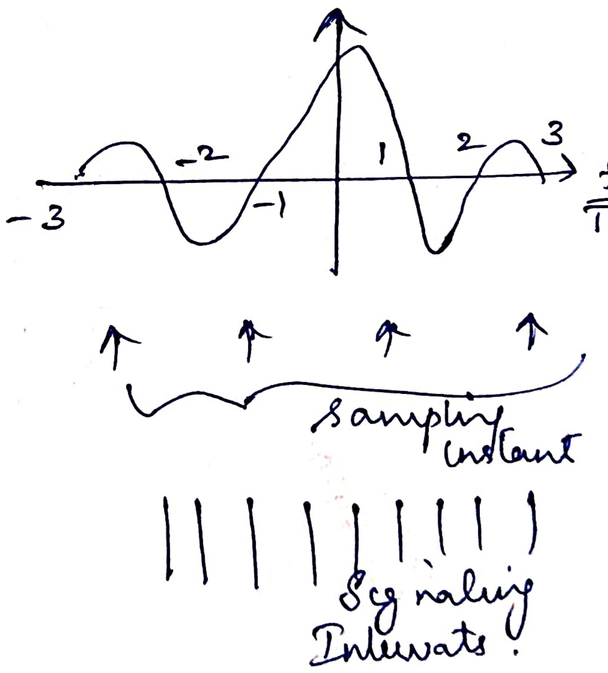
* To Determine Transfer function of Transmitted & Receiving fillers.
* $\{a_k\}$ of o/p $y(t)$
* Scaling factor .
* $t = iT_b$.

$$\boxed{PT\ \delta(t - nT_s) = e^{-2\pi f n T_s}}$$

$$\sum_{n=-\infty}^{\infty} P(f - nR_b) = \frac{1}{R_b} = T_b.$$

1) Ideal solution :-

$$\boxed{= \mu P(t - iT_b)}$$



UNIT-III

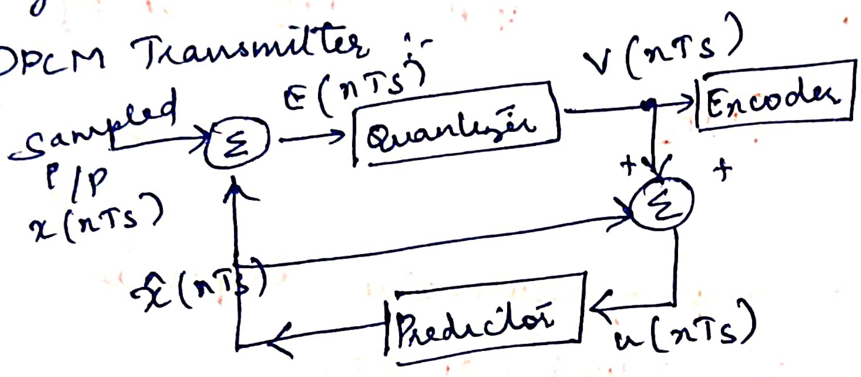
Digital Techniques :-

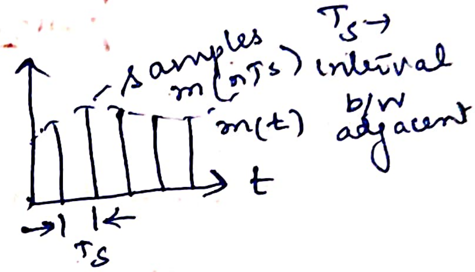
I Pulse modulation Differential Pulse code modulation :-

* Sampled Signal is then found to exhibit a high correlation between adjacent samples.

$f_s = \frac{1}{T_s}$

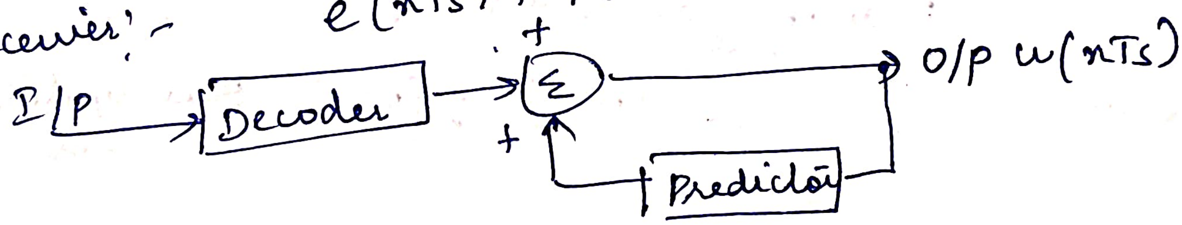
DPCM Transmitter :-





* $e(nT_s)$ → Prediction error.

Receiver :-

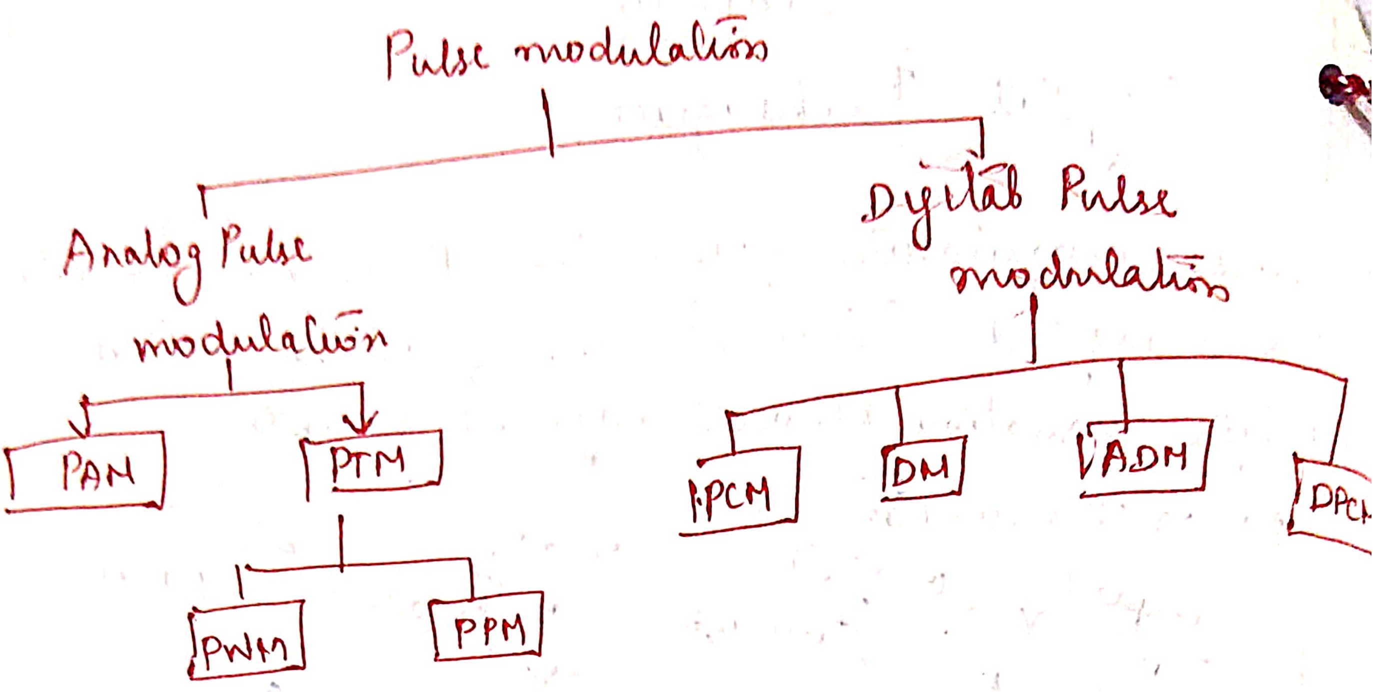


SNR of DPCM :-
$$(SNR)_o = \frac{\sigma_X^2}{\sigma_Q^2}$$

$$(SNR)_o = \left(\frac{\sigma_X^2}{\sigma_\varepsilon^2}\right) \times \left(\frac{\sigma_\varepsilon^2}{\sigma_Q^2}\right)$$

$$= G_p (SNR)_p.$$

$$\boxed{G_p = \frac{\sigma_X^2}{\sigma_\varepsilon^2}.}$$



DPCM :-

Difference in the amp. of 2 successive samples in Transmitted rather than actual sample.

DPCM Tx :- i/p Signal to the Quantizer :-

$\hat{m}[nT_s] \rightarrow$ Prediction of next samples

$$e(nT_s) = m[nT_s] - \hat{m}[nT_s] \quad \text{— (1)}$$

Prediction error signal, sample s/g

quantizer o/p: $e_q[nT_s] = e[nT_s] + q[nT_s]$ — Quantization error. — (2)

* Quantizer o/p $e_q[nT_s]$ is added to Predicted Value $\hat{m}[nT_s]$ to produce the Prediction - Filter i/p :

$$m_q[nT_s] = \hat{m}[nT_s] + e_q[nT_s] \quad \text{— (3)}$$

Sub $e_q[nT_s]$ eqn (2) into (3),

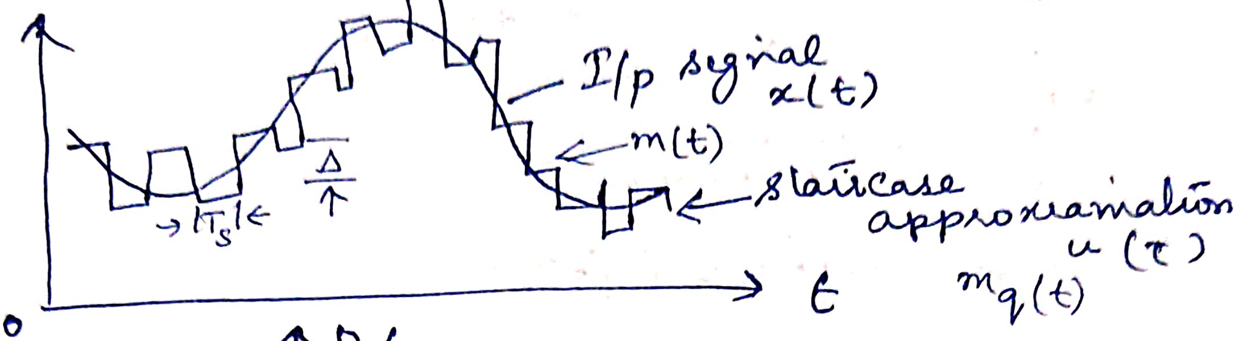
$$m_q[nT_s] = \hat{m}[nT_s] + e[nT_s] + q[nT_s] \quad \text{— (4)}$$

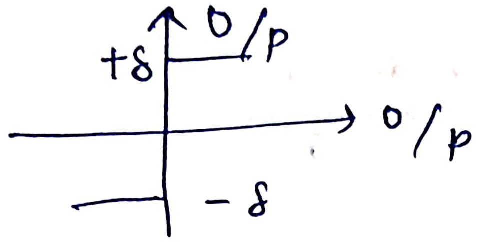
$\therefore$ eqn (1),

$$m_q[nT_s] = m[nT_s] + q[nT_s]$$

Delta Modulation :-

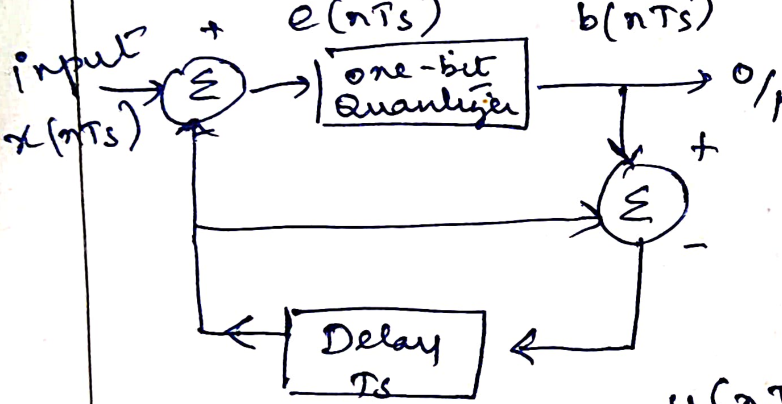
* Delta Modulation is one-bit version of DPCM



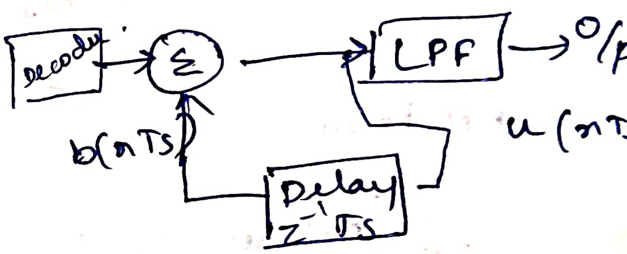


Prediction error, $e(nT_s) = x(nT_s) - \hat{x}(nT_s)$

DM Transmitter



DM Receiver



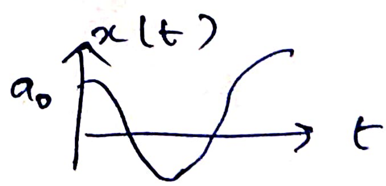
$$u(nT_s) = \sum_{i=1}^{n} b(iT_s)$$

1) Quantization Noise :-

1) Slope overload distortion — $\frac{\delta}{T_s} \geq \max\left|\frac{dx(t)}{dt}\right|$

2) Granular Noise

2) Maximum O/p SNR of DM.



$$\boxed{(SNR_o)_{Max} = \frac{3}{8\pi^2 f_0^2 T_s^3 \cdot W}}$$

DM Transmitter :-

$$m[t] = m[nT_s] \;, n = 0, \pm, \pm 2 \quad \text{--- (1)}$$

msg s/g — $m[t]$

stair case approximation — $m[nT_s]$

$$e[nT_s] = m[nT_s] - \hat{m}[nT_s] \quad \text{--- (2)}$$

error — $e[nT_s]$

sampled s/g $m(t)$ — $m[nT_s]$

staircase waveform — $\hat{m}[nT_s]$

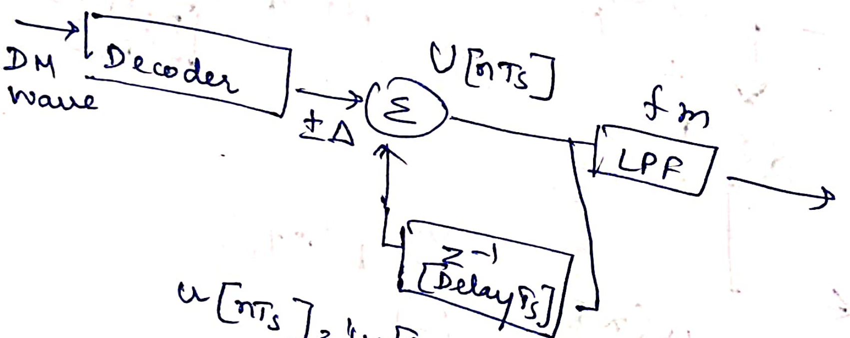
$+\Delta$ binary 1 Tx

$$b[nT_s] = +\Delta \;, \quad m[nT_s] \geq \hat{m}[nT_s]$$

& $-\Delta$ binary 0, Tx

$$b[nT_s] = -\Delta \;, \quad m[nT_s] < \hat{m}[nT_s]$$

DM Receiver :-



$$u[nT_s] = u[(n-1)T_s] + [\pm\Delta]$$

& $+\Delta \rightarrow$ binary '1'

& $-\Delta \rightarrow$ binary '0'.

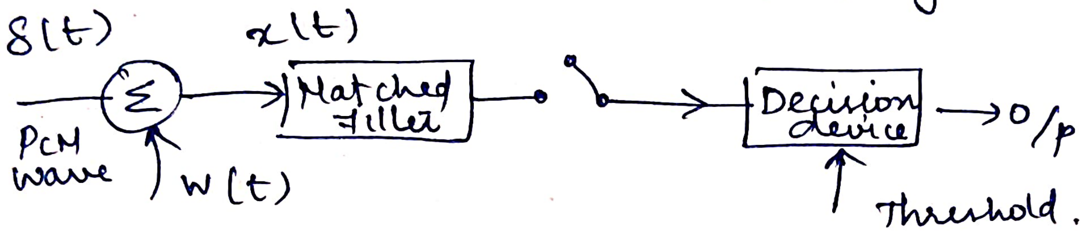
Noise Consideration in PCM :-

1. Channel Noise — Transmission Path.
2. Quantization Noise → Receiver.

* Error rate,
* symbol '1' $s(t) = s_1(t)$.

$$s_1(t) = \sqrt{\frac{E_{max}}{T_b}}, \quad 0 \le t \le T_b.$$

* Received sample $s(t) = \begin{cases} s_1(t), & \text{symbol 1} \\ s_2(t), & \text{symbol 0} \end{cases}$



1st basic fn $\phi_1(t) = \sqrt{\frac{1}{T_b}}, \quad 0 \le t \le T_b$

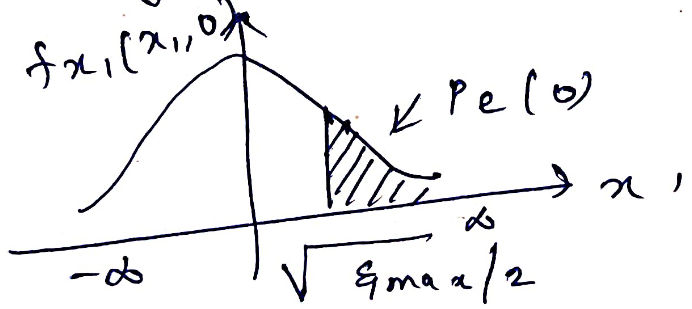
Transmitted waveform,

$$s_1(t) = \sqrt{E_{max}} \cdot \phi_1(t).$$

$$s_{11} = \sqrt{E_{max}}$$

$$s_{21} = 0.$$

* Decision region $Z_1 : \sqrt{\frac{E_{max}}{2}} < x_1 < \infty$

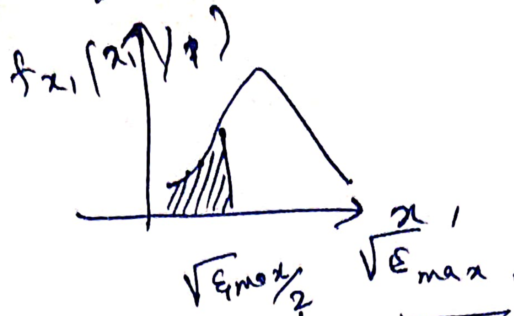


$$P_e(0) = \frac{1}{\sqrt{\pi N_0}} \int_{\sqrt{\frac{E_{max}}{2}}}^{\infty} \exp\left(-\frac{x_1^2}{N_0}\right) dx_1$$

Complementary error function

$$P_e(0) = \frac{1}{2} \operatorname{erfc}\left(\frac{1}{2}\sqrt{\frac{E_{max}}{N_0}}\right)$$

$f_{X_1}(x_1|1)$



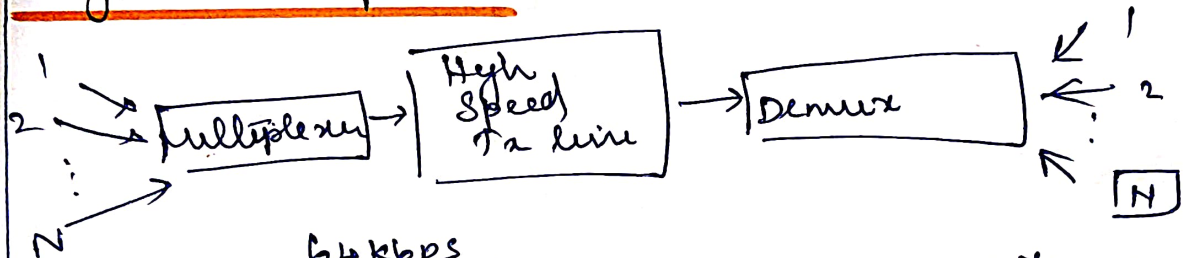
$$P_e(1) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{1}{2}\sqrt{\frac{E_{max}}{N_0}}\right)$$

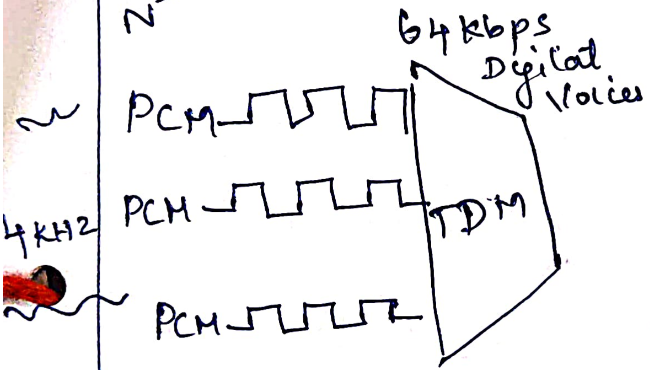
Avg. Probability of error.

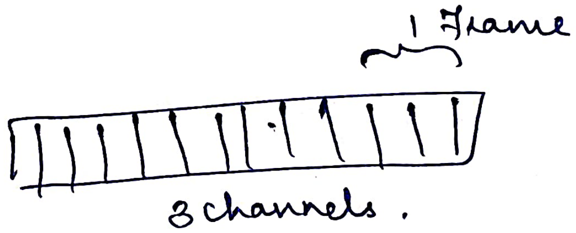
$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\frac{1}{2}\sqrt{\frac{E_{max}}{N_0}}\right)$$

$$\frac{E_{max}}{N_0} = \frac{P_{max}}{N_0/T_b}.$$

IV Digital Multiplexers :-





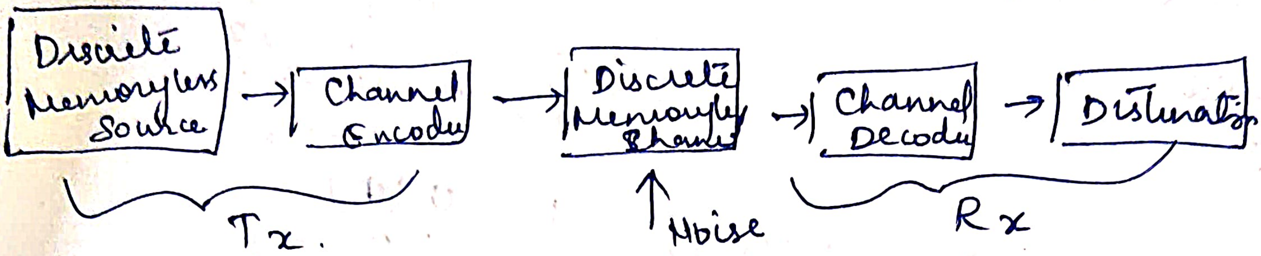


1) Digital TDM Hierarchy :-

1) 1st level → 1.544 Mb/s
2) 2nd level → 6.312 Mb/s
3) 3rd level → 44.736 Mb/s
4) 4th level → 274.176 Mb/s
5) 5th level → 560.160 Mb/s.

Channel coding Theorem:-

* Probability of error $\rightarrow 10^{-6}$.
* Mapping operation — encoder, inverse mapping operation → decoder.



* Ratio K/n called code rate

$$\boxed{r = \frac{K}{n}}$$

i) Discrete Memoryless source,

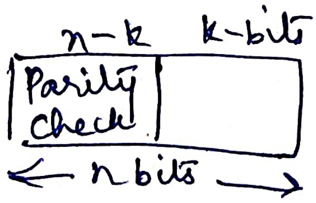
$$\frac{H(\tau)}{T_s} \leq \frac{C}{T_c}.$$

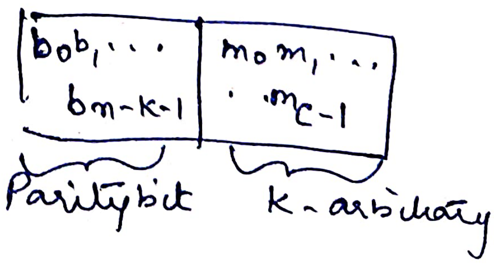
ii) Conversely, $\frac{H(\tau)}{T_s} > \frac{C}{T_c}$ → fundamental limit

Source coding → Reduces Redundancy to ↑ve efficiency
channel coding — introduces Redundancy to ↑se reliable

Linear Block codes:-

* (n-k) bits → generalized parity check bits.
* msg bits are transmitted in unaltered form called systematic code.





$$P_{ij} = \begin{cases} 1, & b_i \text{ depends on } m_j \\ 0, & \text{otherwise}. \end{cases}$$

* (n,k) linear block codes.

$$P = \begin{bmatrix} p_{00} & p_{10} & p_{20} & \cdots & p_{n-k-1,1} \\ p_{01} & p_{11} & p_{21} & \cdots & p_{n-k-1,k-1} \\ \vdots & & & & \\ p_{0,k-1} & p_{k-1} & \cdots & & p_{n-k-1,k-1} \end{bmatrix}$$

$$\boxed{X = mG}$$

Parity check matrix (H)

$$H = [I_{n-k-1} \mid P^T]$$

$$HG^T = P^T \oplus P^T = 0.$$

1) Problems in linear blockcodes.

2) syndrome decoding.

$$\boxed{Y = X + e}$$

$$e_i = \begin{cases} 1, & \text{error occured in } i\text{th location} \\ 0, & \text{otherwise.} \end{cases}$$

VII Hamming codes:-

* (n, k) linear blockcode follow parameters.
  - Block length $n = 2^m - 1$
  - no. of msg bits $k = 2^m - m - 1$
  - no. of parity bits $n - k = m$.
  - $m \geq 3 \rightarrow$ Hamming code.

Problems:

VIII Cyclic codes:-

* cyclic code is a subclass of block code.
* Correct more than one error.

Definition:- i) 'c' is linear code

ii) any cyclic shift of code also a codeword

$\boxed{x_{n-1}, x_{n-2} \ldots x_1, x_0}$ ← original code word

↓ Shift left by 1 bit

$\boxed{x_{n-2} \ldots x_1, x_0, x_{n-1}}$ new code word x_1

↓

$\boxed{x_{n-3}, x_{n-4} \ldots x_1, x_0, x_{n-1}}$ new code word x_2

Shifted words

Algebraic Structures of cyclic codes:-

$$X = \{x_{n-1}, x_{n-2} \ldots x_1, x_0\}.$$

$$X(P) = x_{n-1} P^{n-1} + x_{n-2} P^{n-2} + \ldots + x_1 P + x_0$$

Polynomial. degree $(n-1)$.

MSB

$P^0 = LSB$

Generations of non systematic code Vector (a)

Generator Polynomial for a cyclic code:-

K-bit of msg Vector.

$$M(P) = m_{K-1} P^{K-1} + m_{K-2} P^{K-2} + \ldots + m_1 P + m_0$$

$X(P)$ Represents the code word Polynomial

$$X(P) = M(P) \cdot G(P).$$

* $G(P)$ of degree 'q' for (n, K) cyclic code.

$$G(P) = P^q + g_{q-1} P^{q-1} + \ldots + g_1 P + 1.$$

Generation of systematic Code Vectors:-

$X = (K$ msg bit : $(n-K)$ checkbits) checkbits,

$C(P) =$

$= (m_{K-1}, m_{K-2} \ldots m_1 m_0 : c_{q-1} c_{q-2} \ldots c_1 c_0)$

Book. check bit Polynomial $C(P) = \text{rem}\left[\dfrac{P^q M(P)}{G(P)}\right]$ generator Polynomial

Properties of cyclic codes:-

1. Linearity property:- → Sum of any 2 code words also is a code word

Sum of 2 code word,

	000100
C_1	101011
C_2	101111

$x_3 = x_1 \oplus x_2$

(35)

2. Cyclic Property:- cyclic shift of a code is also a codeword

C_2 111101. → cyclic shift of a valid code vector produces another valid code vector.

Types: systematic (msg unaltered)

non-systematic (msg altered).

$$X(D) = X_0 D^0 + X_1 D^1 + \dots + X_{n-1} D^{n-1}$$

D^0 → LSB, D^{n-1} → MSB.

$X = \{x_{n-1}, x_{n-2}, \dots x_1, x_0\}$

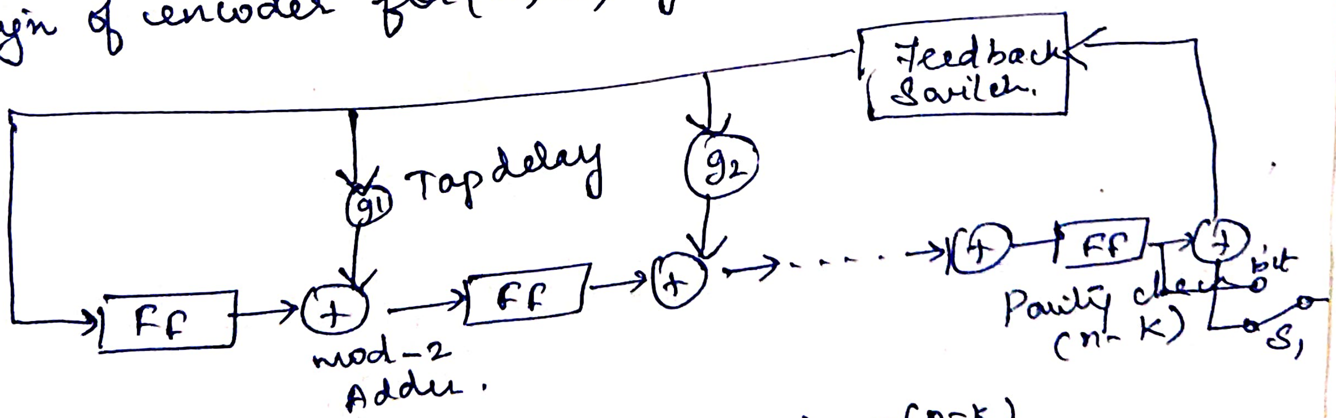
1) Non-systematic cyclic code:-

$$\boxed{x(D) = m(D) \times g(D)}$$

2) systematic code:-

1. multiply $m(D)$ by D^{n-k}
2. Divide $m(D) \times D^{n-k}$ by $g(D)$.
3. Codeword, $x(D) = m(D) \times D^{n-k} + b(D)$

3) Design of encoder for (n,k) cyclic code.



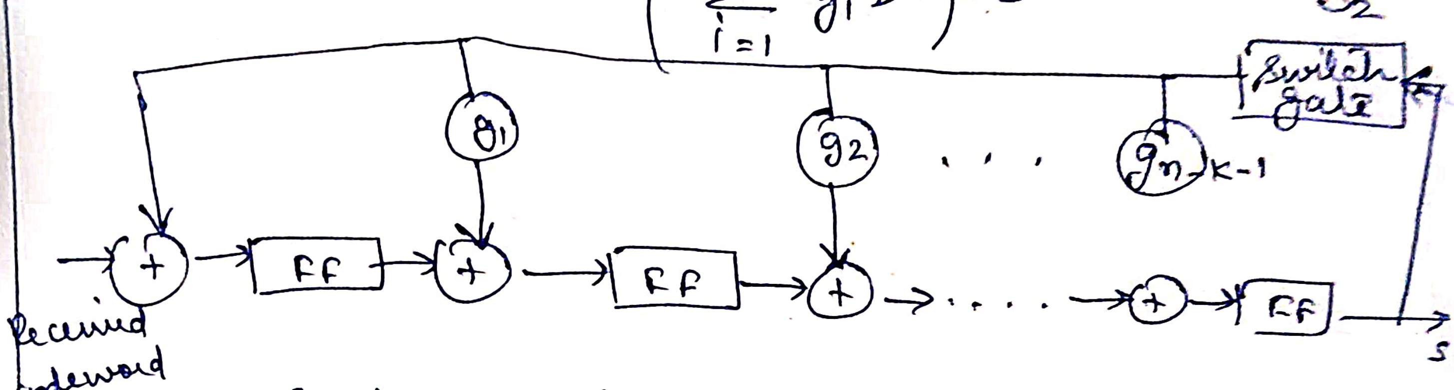
* Systematic cyclic code $\dfrac{m(D) \times D^{(n-k)}}{g(D)}$.

Implementation steps :-

$$g(D) = 1 + \sum_{i=1}^{n-k-1} g_i D^i + D^{n-k}.$$

4) Design of syndrome calculator for a (n,k) cyclic code:-

1. no. of flip flops $(n-k)$
2. Generator $g(D) = 1 + \left(\sum_{i=1}^{n-k-1} g_i D^i\right) + D^{n-k}.$



Syndrome Calculator (n,k) cyclic code.

Vector $[S]_{1\times(n-k)}$.

5) Syndrome for error correction:-

error Detection $[S] = \text{rem}\left[\frac{Y(D)}{g(D)}\right]$

" correction $[S]\,\text{rem}\left[\frac{e(D)}{g(D)}\right]$

6) Other types of cyclic codes:-

1. cyclic Redundancy check (CRC)
2. Golay code.
3. BCH code
4. Reed - solomon code.

IX) **Convolutional codes:-**

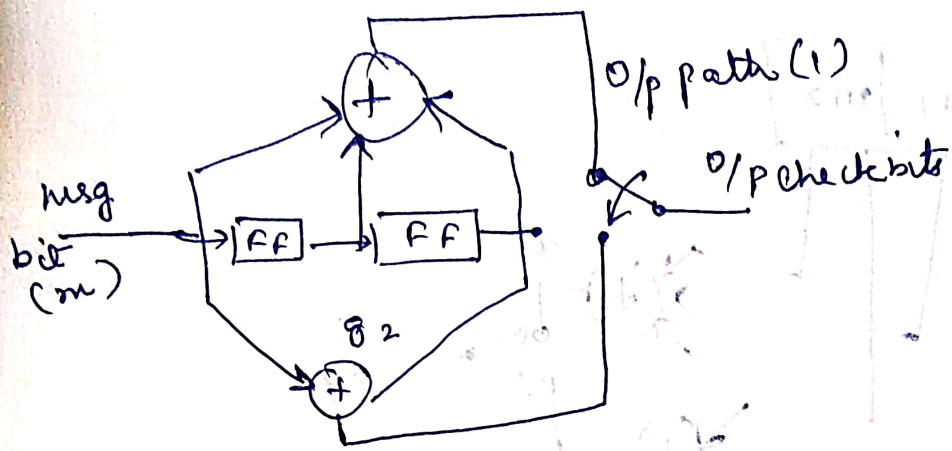
- Another type of error - correcting code.
- msg bits comes serially rather the blocks.
- Represented by (n, k, K)

Code rate, r :- $r = \dfrac{\text{no. of bits in msg}}{\text{Codeword length}}$

$r = \frac{1}{n}$ bits / symbol.

Applications of Convolutional Codes:-

$K = M + 1$



1. Time domain approach:-

$$x_i^{(n)} = \sum_{l=0}^{M} g_l^{(n)} m_{i-l}$$

2. Transform domain approach:-

$$g^{n}(D) = \{g_0 D^0 + g_1 D^1 + \dots + g_m D^m\}$$

3. Code Tree :

4. Trellis Method.

Viterbi Decoding Algorithm:-

* equivalent b/w max. likelihood decoding & minimum distance decoding for a binary symmetric channel implies that we may decode a convolution code.
* code tree is equivalent to trellis.

1) Viterbi Decoder Implementation Steps :-

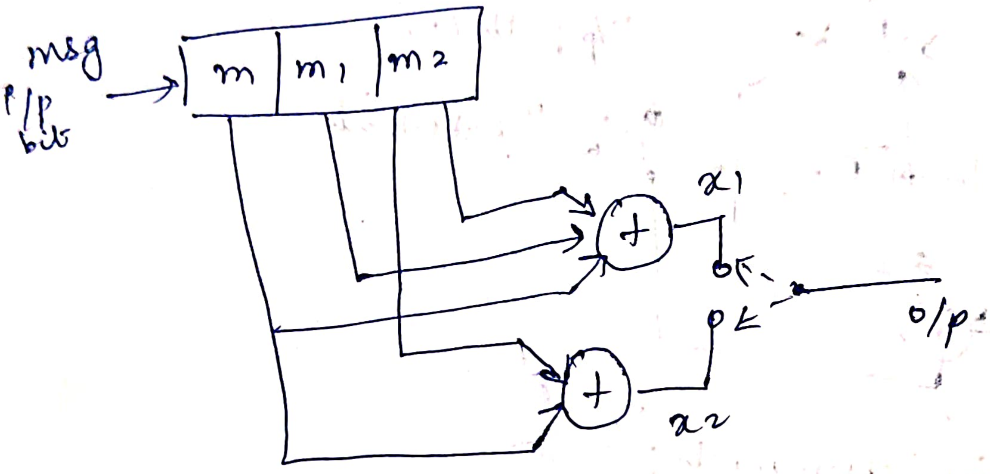
2) Free distance (d free)

3) Modified state diagram.

Convolutional code:-

$X_1 = m_0 \oplus m_4 \oplus m_2$

$X_2 = m \oplus m_2$.



Con.c. is done by combining the fixed no. of i/p bits. The i/p bits are stored in the fixed length shift register & they are combined with the help of modulo-2 adders.

Code Rate(r)

$r = \left[\frac{k}{n}\right] = \frac{1}{2}$

UNIT–IV

Digital modulation scheme:

I Geometric Representation of signals:-

* $\{s_i(t)\}$, $i = 1, 2 \dots M$ set of orthonormal basis function.

coefficient 's_{ij}' $= \int_0^T s_{ij}(t) \cdot \phi_j(t) \cdot dt$, $\quad i = 1, 2 \dots M$, $\; j = 1, 2 \dots N$

energy s/g

orthonormal basis function

$$S_i = \begin{bmatrix} s_{i1} \\ s_{i2} \\ \vdots \\ s_{iN} \end{bmatrix} \qquad N \leq M.$$

$$= \int_0^T \phi_i(t) \cdot \phi_j(t) \cdot dt$$

$$= \delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & i \neq j \end{cases}$$

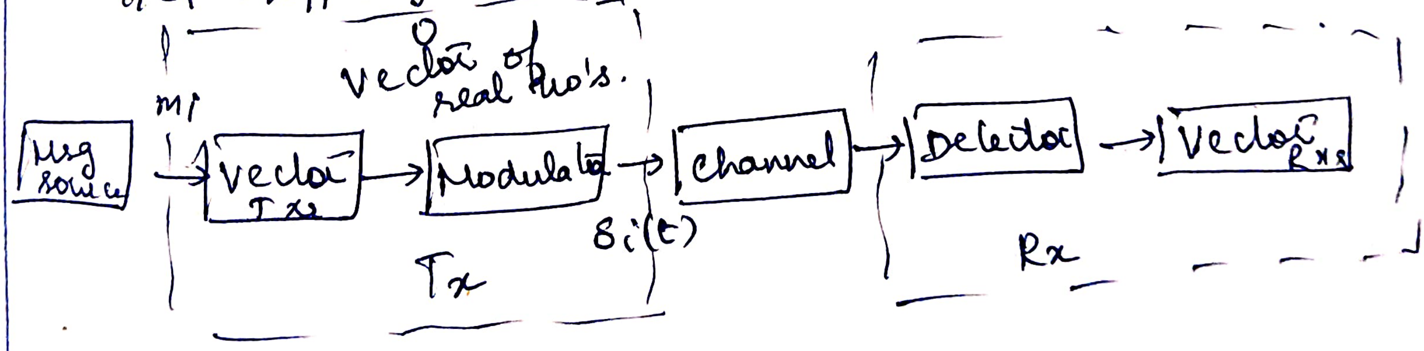
Kronecker delta

* Length or norm $\rightarrow \|s_i\|$.
* Squared length.

$$E_i = \int_0^T s_i^2(t) \cdot dt.$$

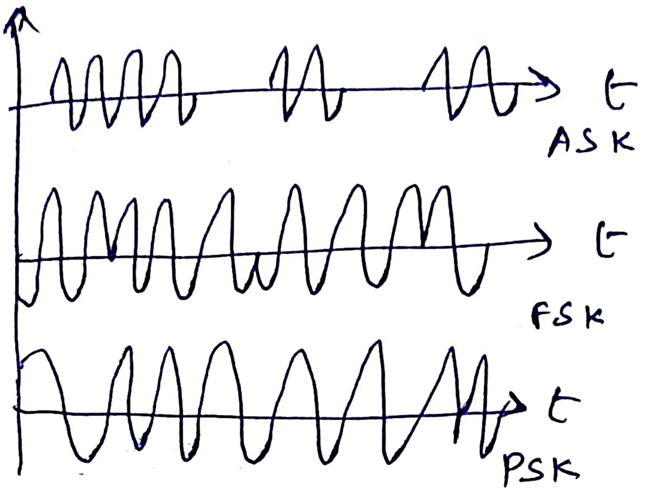
$$E_i = \sum_{j=1}^{N} \sum_{k=1}^{N} s_{ij} \, s_{ik} \int_0^T \phi_j(t) \cdot \phi_k(t) \, dt.$$

$$\|s_i - s_k\|^2 = \int_0^T [s_i(t) - \phi_k(t)]^2 dt.$$



Generation & Detection, IQ Representation

- Amplitude Shift Keying (ASK)
- Frequency shift keying [FSK]
- phase shift Keying [PSK].



1. ASK :- $S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_1 t$, symbol 1

   $S_2(t) = 0$, symbol 0

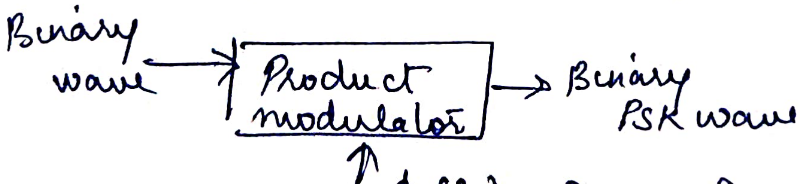
2. FSK :- $S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_1 t$, symbol 1

   $S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_2 t$, symbol 0.

3. PSK :- $S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t$, symbol 1

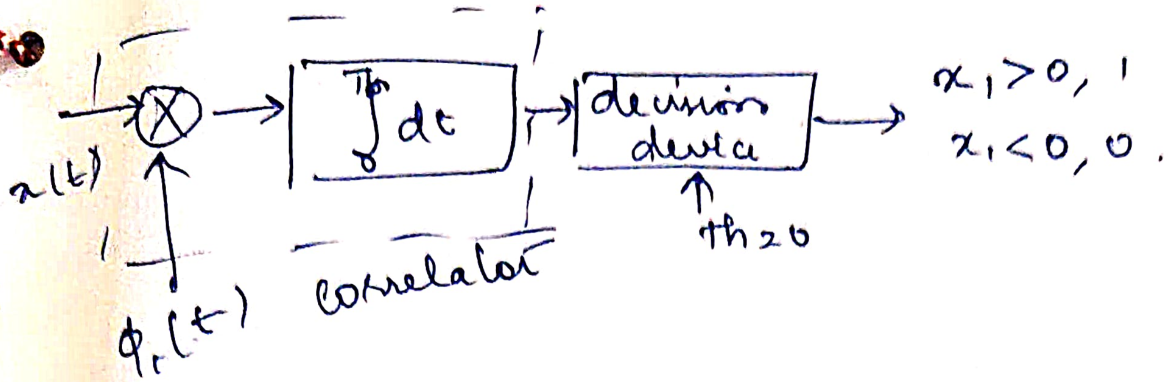
   $S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t$. , S 0.

1) Generation & detection of binary PSK :-



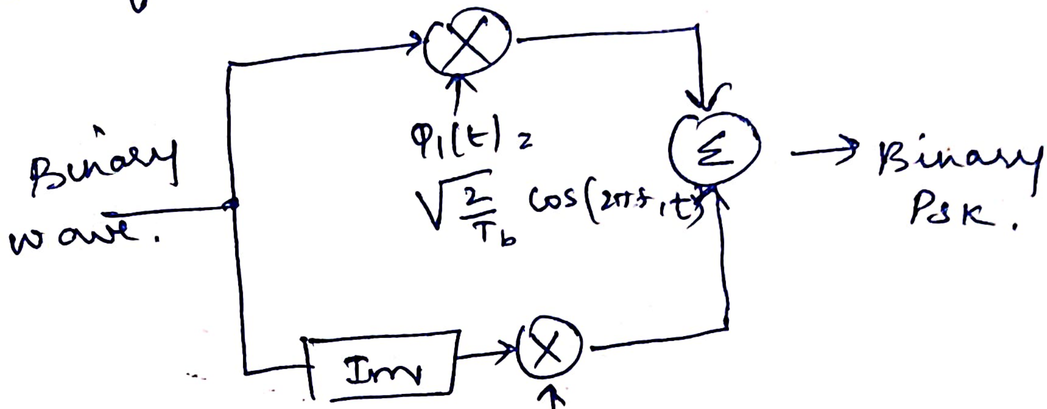
$\phi_1(t) = \frac{2}{T_b} \cdot \cos(2\pi f_c t)$

Rx :-

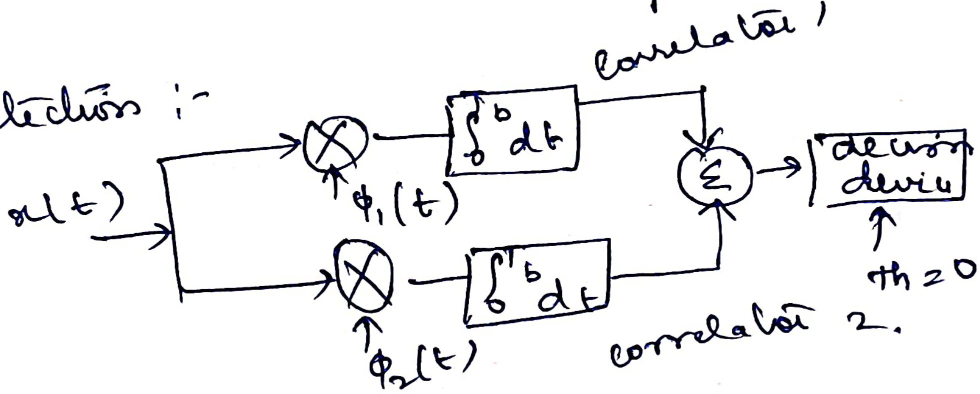


2) Generation and detection of binary FSK.

i) Generation of BFSK.



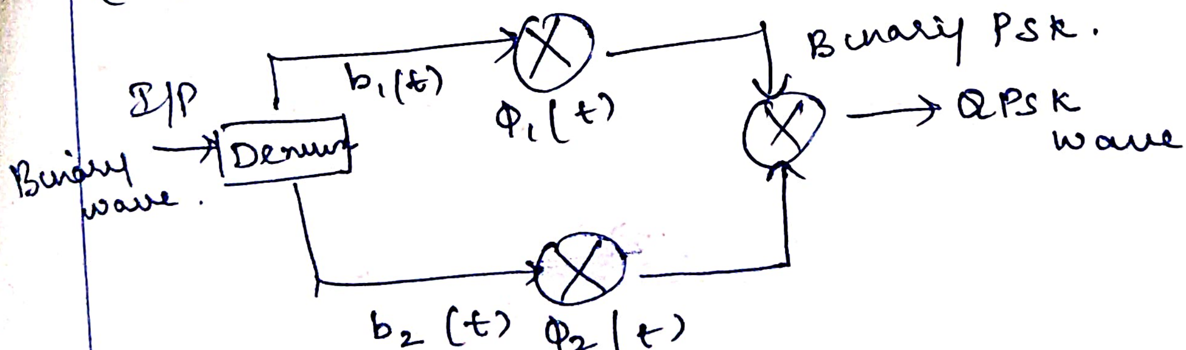
ii) Detection :-



3) Generation & demodulation of QPSK.

* Dibits.

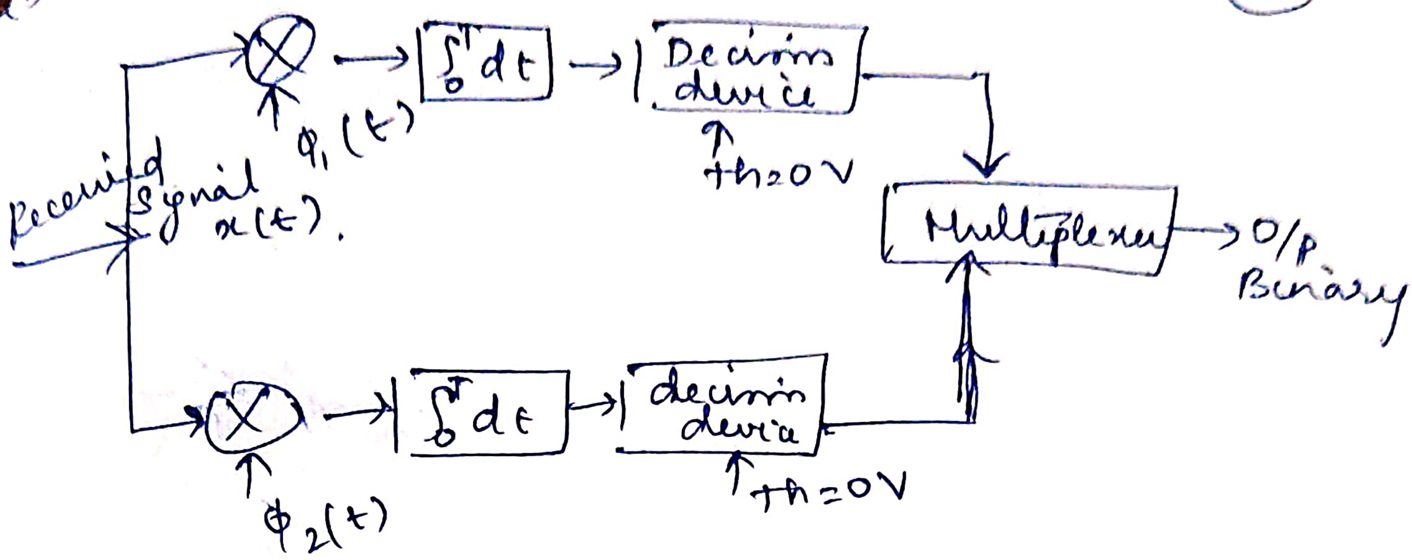
i) QPSK Transmitter :-



$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$$

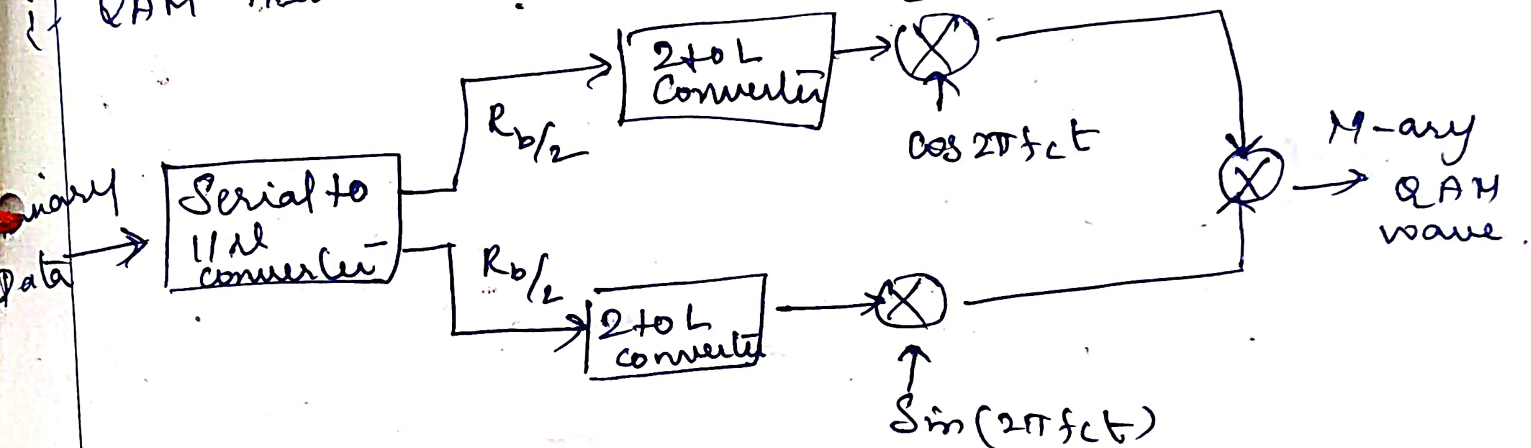
$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$$

iii) Receiver :-

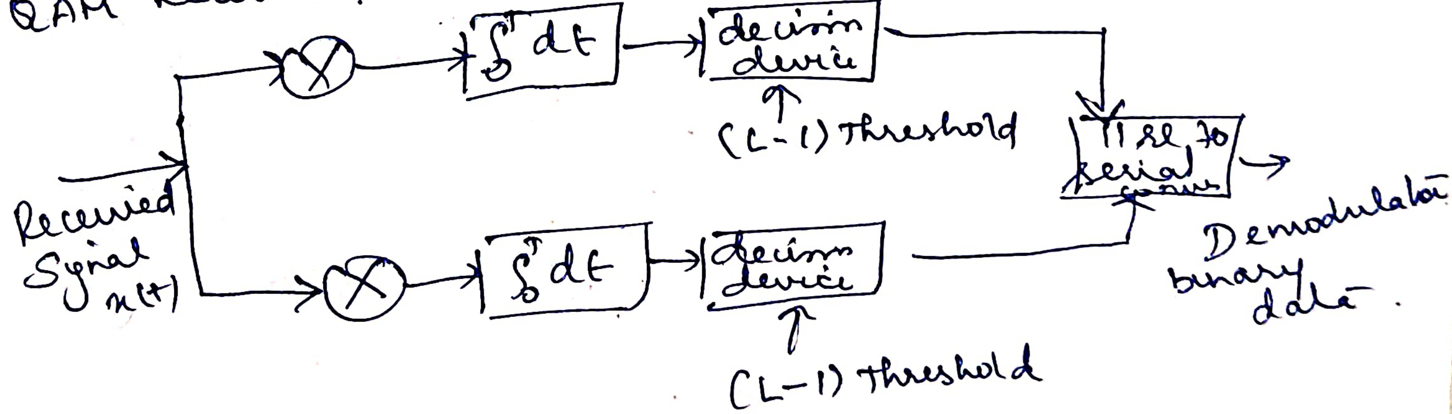


4) M-array QAM Tx & Rx Transmitter.

i) QAM Transmitter :-



ii) QAM Receiver :-



* (L-1) decision Thresholds :

PSD of BPSK, BFSK, QPSK & QAM:-

1) Power spectra of binary PSK & FSK signals.

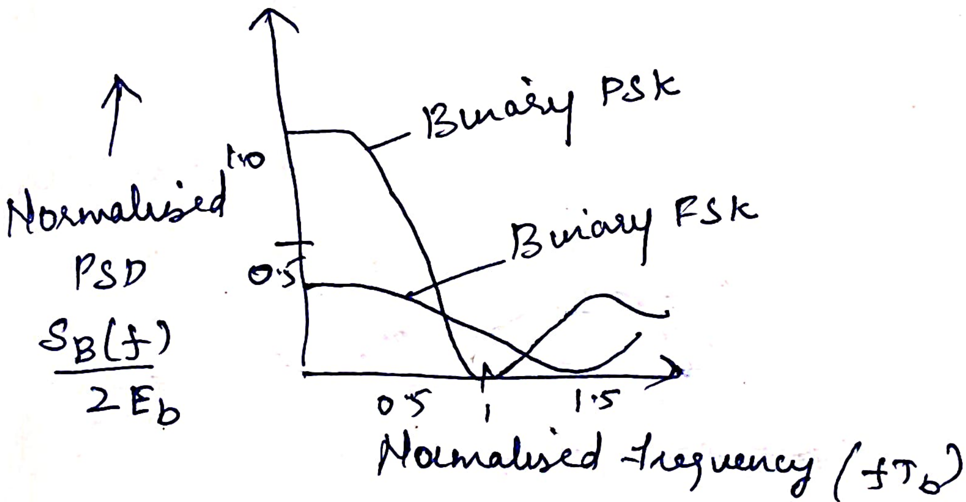
i) Binary PSK:

$$g(t) = \begin{cases} \sqrt{\frac{E_b}{T_b}}, & 0 \le t \le T_b \\ 0, & \text{elsewhere.} \end{cases}$$

$$\boxed{S_B(f) = 2E_b \operatorname{sinc}^2(T_b f)}$$

ii) Binary FSK:

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(\frac{\pi t}{T_b}\right) \cos(2\pi f_c t) \pm \sqrt{\frac{2E_b}{T_b}} \sin\left(\frac{\pi t}{T_b}\right) \sin(2\pi f_c t)$$



$$\Psi_g(f) = \frac{8 E_b T_b \cos^2(\pi f T_b)}{\pi^2 (4T_b^2 f^2 - 1)^2}$$

$$S_B(f) = \frac{E_b}{2T_b}\left[\delta\left(f - \frac{1}{2T_b}\right) + \delta\left(f + \frac{1}{2T_b}\right)\right] + \frac{8 E_b T_b \cos^2(\pi f T_b)}{\pi^2 (4T_b^2 f^2 - 1)^2}$$

2) PSD of QPSK Signal:-

$$g(t) = \begin{cases} \sqrt{\frac{E}{T}}, & 0 \le t \le T \\ 0, & \text{elsewhere.} \end{cases}$$

$$S_B(f) = 4E_b \operatorname{sinc}^2(2T_b f)$$

BER of coherent BPSK, FSK, QPSK & QAM.

(Bit error rate / Probability of error).

Coherent binary PSK:-

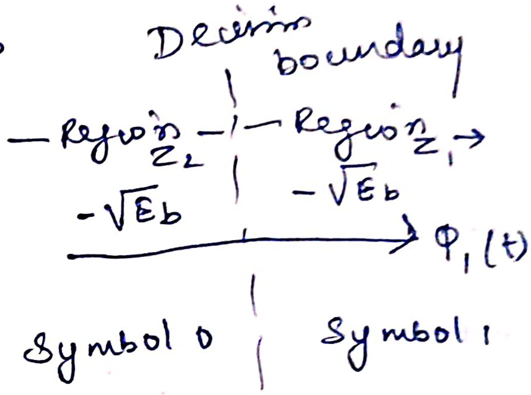
$$S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$$

$$S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi)$$

msg points equal to,

$$S_{11} = \int_0^{T_b} S_1(t) \cdot \phi_1(t) = \sqrt{E_b}$$

$$S_{21} = \int_0^{T} S_2(t) \cdot \phi_1(t) = -\sqrt{E_b}$$



* conditional Probability,

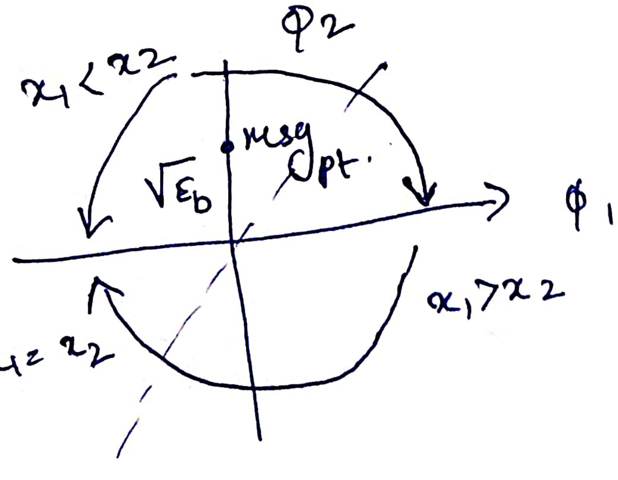
$$P_e(0) = \frac{1}{2} \operatorname{erfc}\left[\sqrt{\frac{E_b}{N_0}}\right]$$

* avg conditional error,

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Coherent binary FSK:-

$$S_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cdot \cos(2\pi f_i t), & 0 \le t \le T_b \\ 0, & \text{elsewhere.} \end{cases}$$



$$x(t) = S_1(t) + w(t)$$

$$x(t) = S_2(t) + w(t)$$

* conditional mean,

$$E[L|0] = -(\sqrt{E_b})$$

* symbol 0 was Transmitted,

$$f_L(l|0) = \frac{1}{\sqrt{2\pi N_0}} \exp\left[-\left(\frac{(l+\sqrt{E_b})^2}{2N_0}\right)\right]$$

* $$P_e(0) = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

* avg probability for coherent binary FSK,

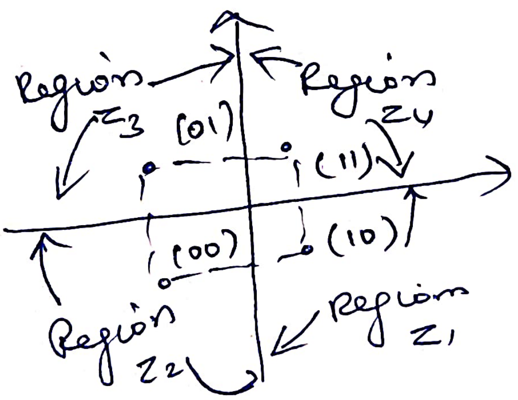
$$P_e = \frac{1}{2}\,\text{erfc}\left(\frac{E_b}{2N_0}\right)$$

3) Coherent Quadrature - modulation techniques :-

1) Coherent QPSK :-

Phase of Carrier $\frac{\pi}{4}, \frac{8\pi}{4}, \frac{5\pi}{4}$ & $\frac{7\pi}{4}$.

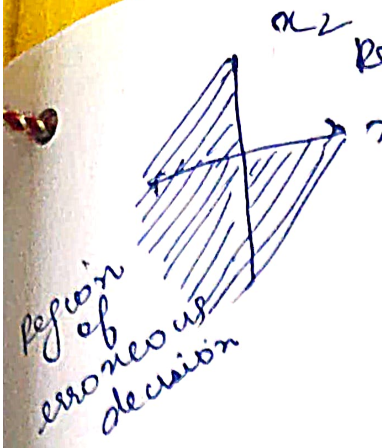
$$S_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + (2i-1)\pi/4\right), & 0 \le t \le T \\ 0, & \text{elsewhere.} \end{cases}$$



$$x_1 = \sqrt{E}\cos(2i-1)\pi/4 + w_1$$

$$x_2 = \int_0^T x(t)\cdot\phi(t)\,dt$$

$$x_2 = -\sqrt{E}\sin\left[(2i-1)\pi/4\right. + w_2$$



* avg probability of symbol error

$$\boxed{\therefore P_e \simeq \mathrm{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)}$$

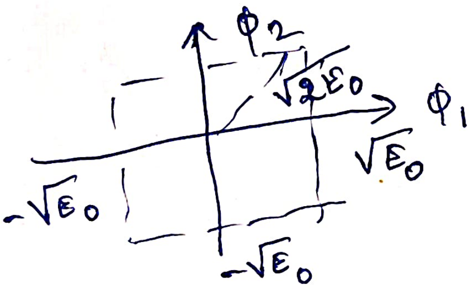
QAM. Quadrature amplitude modulation.

$$\{a_i, b_i\} = \begin{bmatrix} (-L+1, L-1) & (-L+3, L-1) & (L-1, L-1) & (L-1, L-1) \\ \vdots & & & \\ \vdots & & & \\ (-L+1, -L+1) & (-L+3, -L+1) & \cdots & (L-1, L+1) \end{bmatrix}$$

Symbol error probability for M-array QAM.

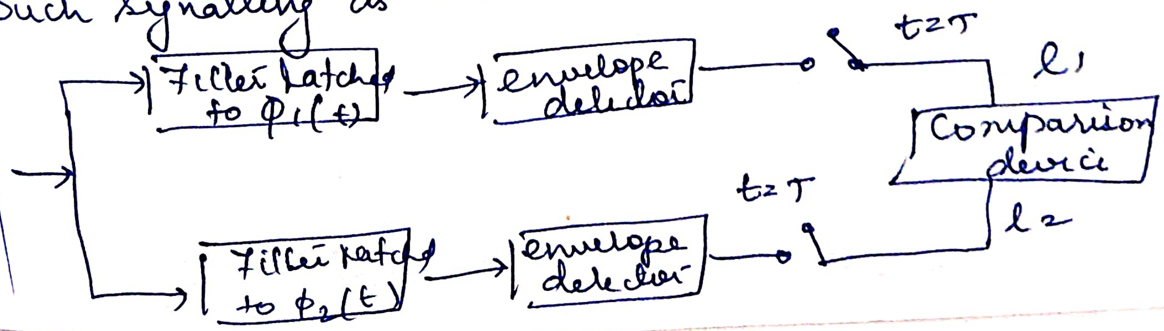
$$E_{av} = 2\left[\frac{2E_0}{L}\sum_{i=1}^{L/2}(2i-1)^2\right]$$

$$P_e = 2\left(1 - \frac{1}{\sqrt{M}}\right)\mathrm{erfc}\left(\sqrt{\frac{3E_{av}}{2(M-1)N_0}}\right)$$

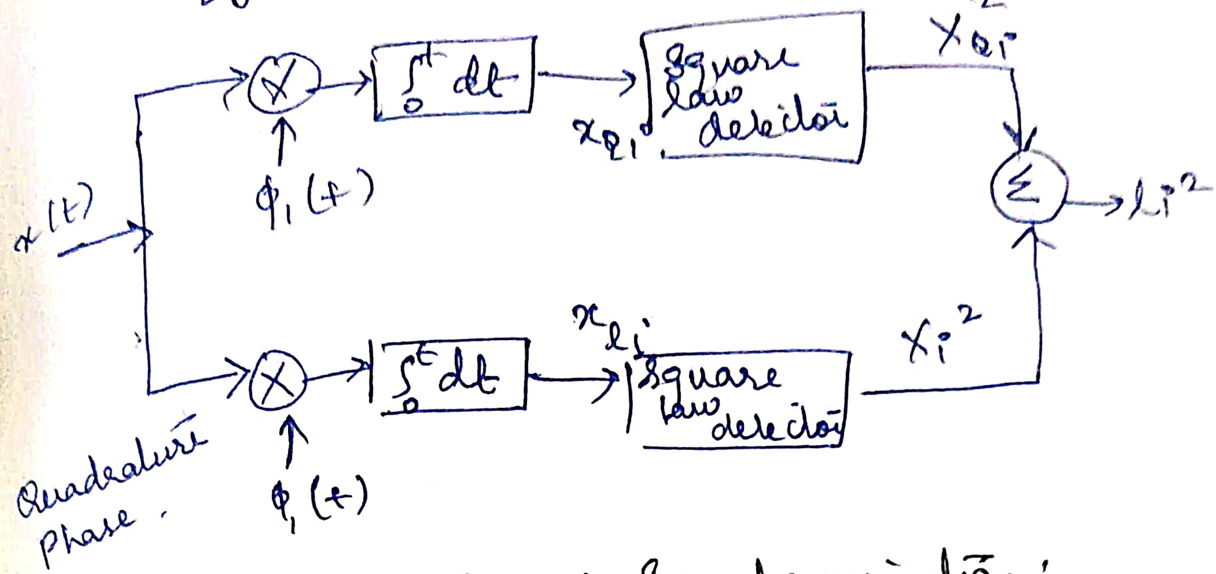


V) Structure of non-coherent Receiver :-

* $g_1(t)$ & $g_2(t)$ are orthogonal regardless of unknown carrier. Such signalling as non-coherent orthogonal modulation.



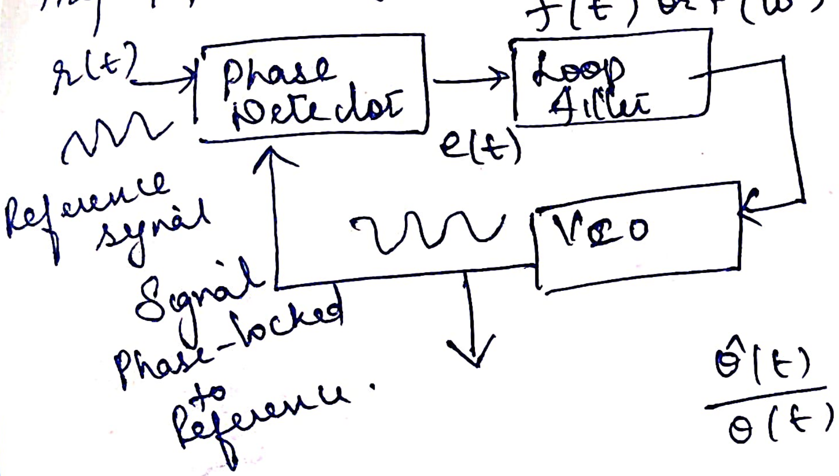
$$x(t) = \begin{cases} g_1(t) + w(t), & 0 \le t \le T \\ g_2(t) + w(t), & 0 \le t \le T. \end{cases}$$



VI) Carrier Recovery or Carrier Synchronization :-

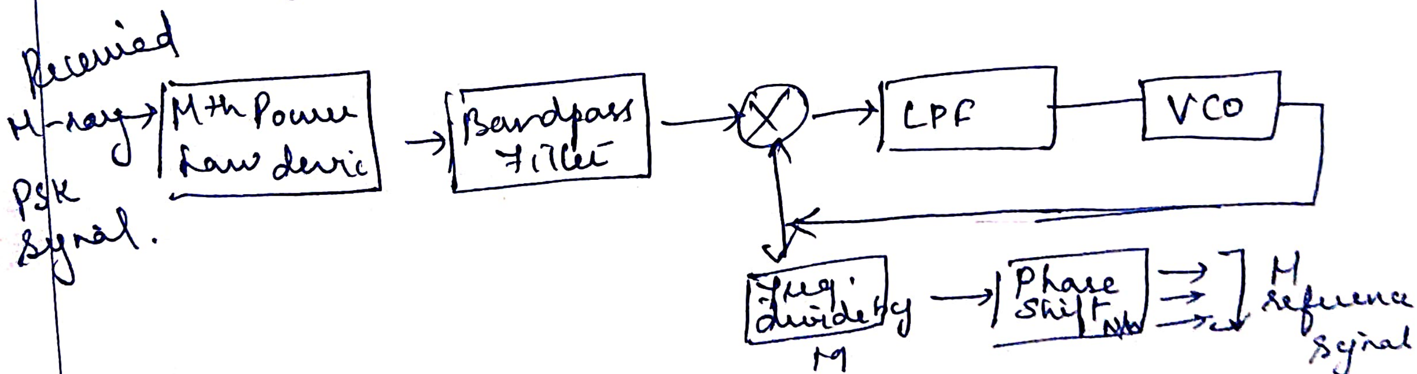
1. Carrier synchronization.
2. Symbol synchronization.

1) Freq. & phase synchronization



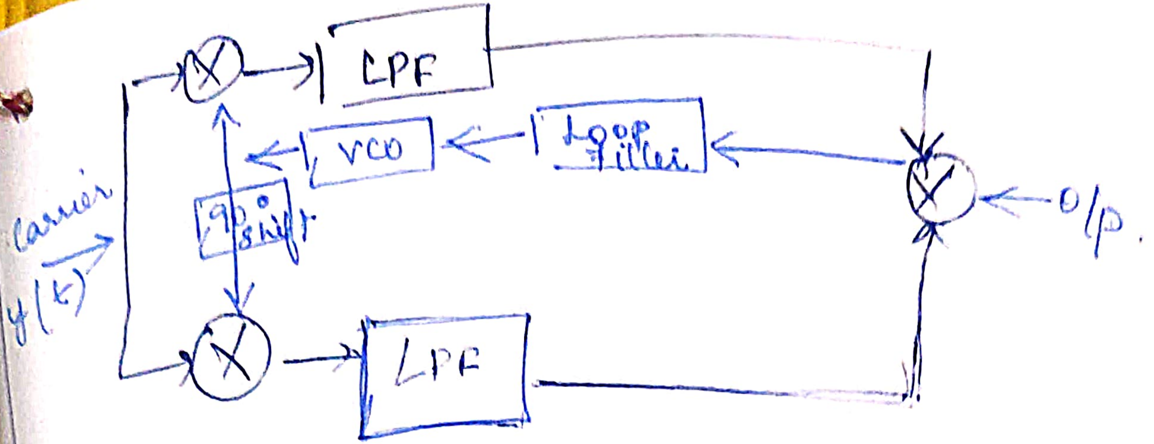
$$\frac{\hat{\theta}(t)}{\theta(t)} = \frac{K_0 F(\omega)}{j\omega + K_0 F(\omega)} = H(j\omega)$$

2) Carrier synchronization using Mth power loop.

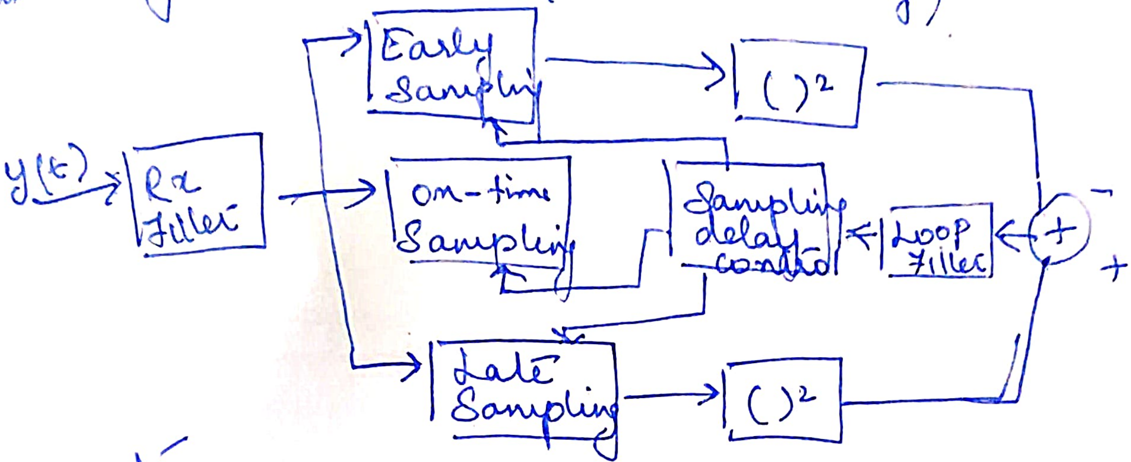


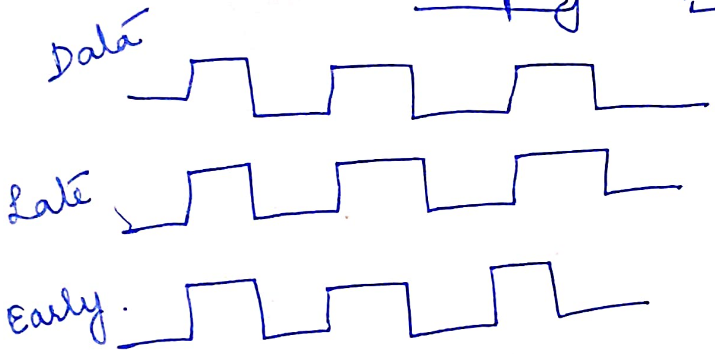
3) Costas loop :-

$P = E/T$



6) Symbol synchronization (clock recovery)





VII Spectrum Analysis:-

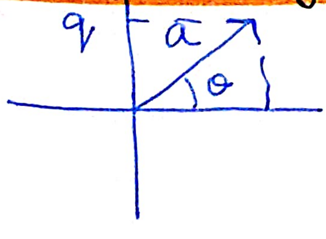
1) Adjacent channel Power. $ACPR_{dBc} = 10 \times \log_{10}\left(\frac{P_{adj}}{P_{ch}}\right)$

2) Calculating channel Power

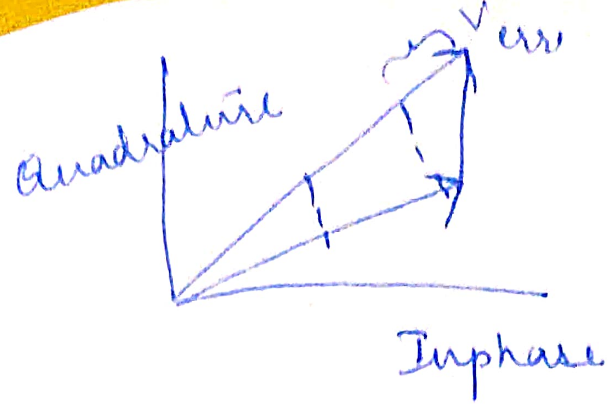
$$P_{ch} = \frac{\sum_f 10\left[\frac{FFT\ Bin(f)}{10}\right]}{\text{window bandwidth}}$$

3) Occupied Bandwidth.

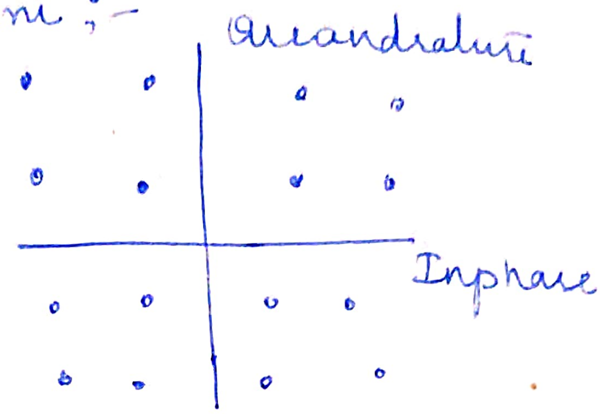
VIII Error Vector Magnitude (EVM):-



* Relative Constellation Error (RCE)



constellation diagram :-



Principle of DPSK :-

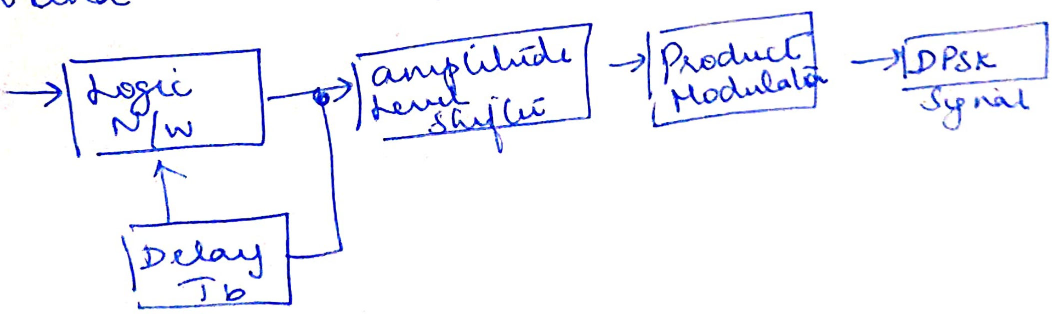
2 basic operation

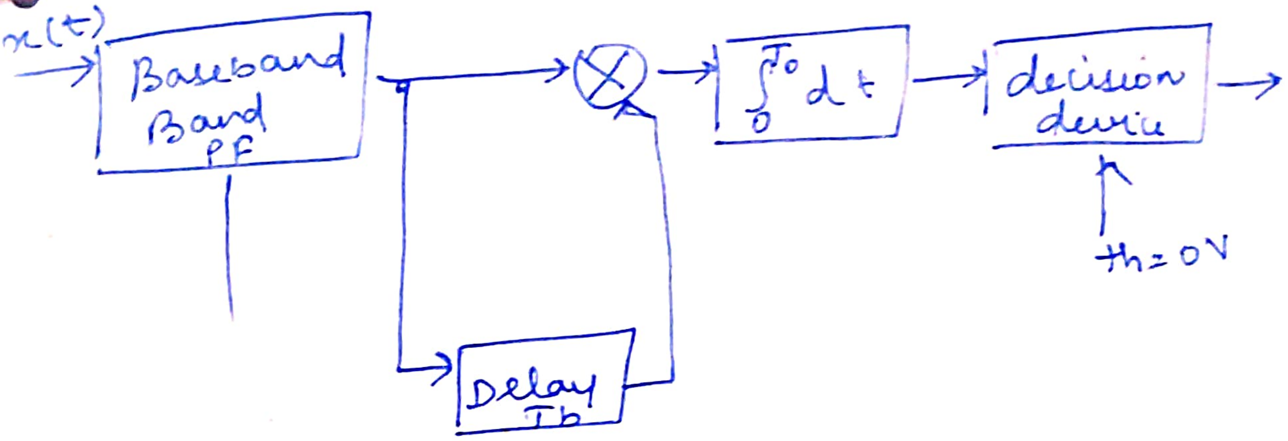
1. Differential encoding of input binary wave
2. phase shift keying.

$$s_1(t) = \begin{cases} \sqrt{\dfrac{E_b}{2T_b}} \cos 2\pi f_c t, & 0 \le t \le T_b \\ \sqrt{\dfrac{E_b}{2T_b}} \cos 2\pi f_c t, & T_b \le t \le 2T_b \end{cases}$$

$$\boxed{P_e = \frac{1}{2} \exp\left(\frac{-E_b}{2N_0}\right)}$$

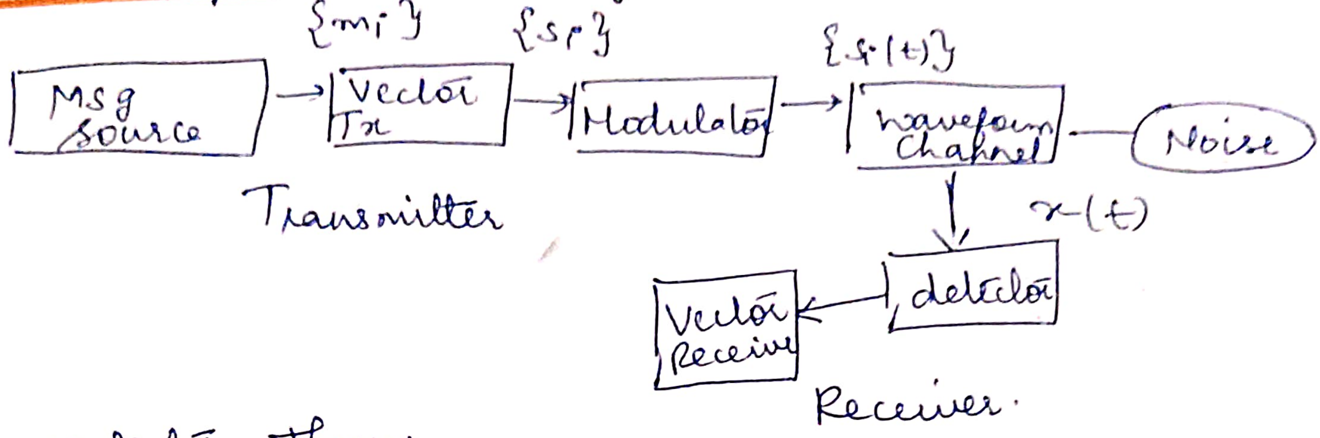
1. Generation and Detection of DPSK.





* carrier phase angles.

I Elements of Detection Theory :-



1. Detection Theory
2. Estimation Theory.

$$P_i = 1/M \text{ for all } i.$$

$$S_i = \begin{bmatrix} S_{i1} \\ S_{i2} \\ \vdots \\ S_{iN} \end{bmatrix}, \quad i = 1, 2 \ldots M, \quad N \leq M.$$

1) Likelihood Ratio Test.

$$\bar{C} = \underset{i,j \in [i,j]}{SIGMA} \; C_{ij} \, \Pi_j \, P_r \left[\text{say } M_i \mid M_j \text{ true} \right]$$

II optimum Detection of signals in noise :-

Far & away the most common decision Problem in signal processing.

$$H_0 : R(l) = S_0(l) + N(l), \quad 0 \leq l < L$$

$$H_1 : R(l) = S_1(l) - N(l), \quad 0 \leq l < L$$

$\{S_i(l)\} \rightarrow$ signals, $N(l) \rightarrow$ stationary stochastic Process.

i) Additive White Gaussian Noise (AWGN)

$$P_N(x - S_i) = \left(\frac{1}{2\pi\sigma^2} \right)^{T} e^{-\left(\frac{1}{2\sigma^2} (x - S_i)^T (x - S_i) \right)}$$

$$\Upsilon_i(x) = x^T S_i$$

Response of bank of correlators to noisy input:-

$$x(t) = \sum_{j=1}^{N} x_j \phi_j(t) + w'(t)$$

Mean & Variance:-

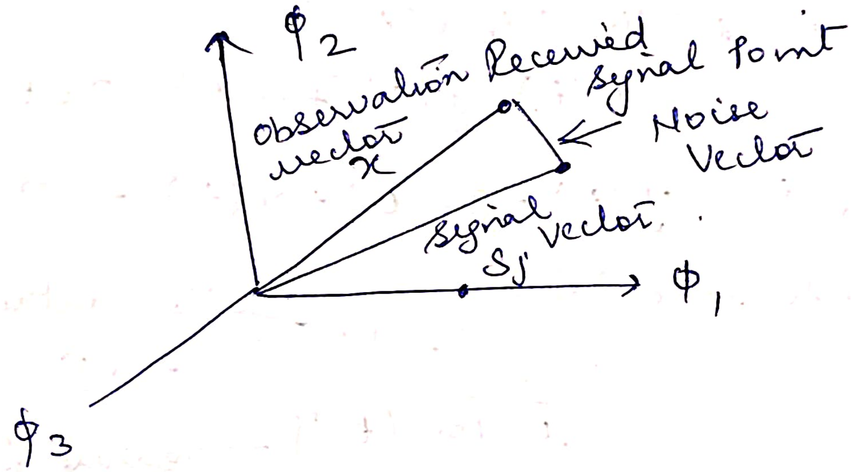
Time diff $R_w(t,u) = \frac{N_0}{2} \int_0^T \phi_j^2(t) \cdot dt$.

$$\text{conv}[x_j x_k] = \int_0^T \int_0^T \phi_j(t)\, \phi_k(u)\, R_w(t,u) \cdot dt \cdot du.$$

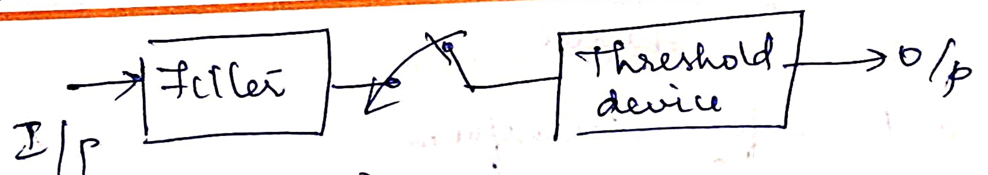
$$= 0, \; j \neq k.$$

Detection of known signals in noise:-

$$x(t) = S_i(t) + w(t), \quad 0 \le t \le T$$
$$i = 1, 2, 3 \ldots M.$$



(ii) Coherent Detection (or) Synchronous Detection



$v(t) = z(t) + n(t)$

Optimum receiver:- yields minimum probability of error.

i) Matched Filter.

ii) Bit Rate & Baud rate.

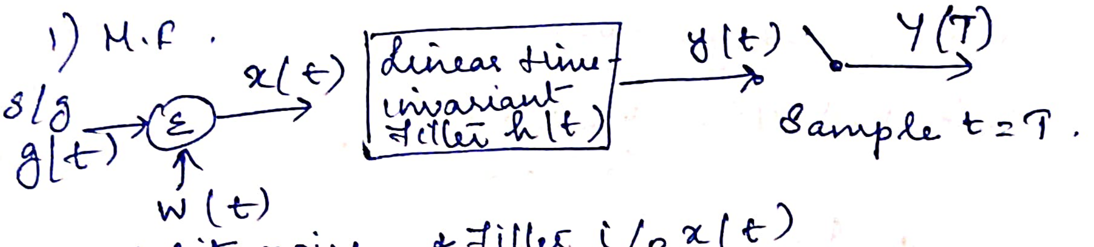
$$\text{Baud rate} = \frac{\text{Bitrate}}{\text{No. of bits per signal unit}}$$

$$= \text{Bitrate} / \log_2 M.$$

Baseband Pulse Transmission :-

* Tr of digital Pulse data over Baseband channel is called B.P.T.
* Baseband channel is typically dispersive, freq. response deviates from an ideal L.P.F. This create interference b/w diff. adjacent Pulse called ISI.
* Bit error during detection is called channel noise.
* Linear-time invariant filter is called matched filter.

1) M.F.



* Filter i/p $x(t)$

$$x(t) = g(t) + w(t), \quad 0 \leq t \leq T \quad \text{---} (1)$$

* o/p $y(t)$, $y(t) = g_0(t) + n(t)$ --- (2)

* Signal to noise ratio

$$\eta = \frac{|g_0(t)|^2}{E[n^2(t)]} \quad \text{---} (3)$$

$|g_0(t)|^2$ — Instantaneous power in o/p s/g.

$E[n^2(t)]$ — measure of avg o/p noise Power.

E — Statistical expectation operator.

* $g_0(t)$ inverse fourier Transform,

$$g_0(t) = \int_{-\infty}^{\infty} H(f)\, G(f) . \exp(j2\pi ft) . df.$$

* Filter o/p Sampled at time $t=T$,

$$|g_0(t)|^2 = \left| \int_{-\infty}^{\infty} H(f) . G(f) . \exp(j2\pi fT) . df \right|^2 \quad (4)$$

* power spectral density, $S_N(f)$,

$$S_N(f) = \frac{N_0}{2} |H(f)|^2 \quad \text{---} (6).$$

IV Probability of error evaluation :-

* Probability of error (P_e) :-
  Ranges 10^{-4} to 10^{-7}
* A factor of $\sqrt{2}$ need to be noticed

$$Q(z) = \int_{z}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy.$$

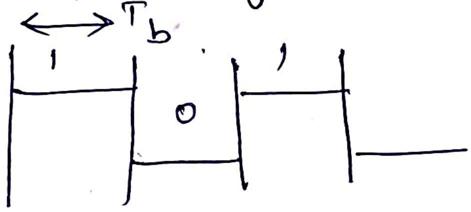
$$= \frac{1}{2} \operatorname{erfc}(\mu) = \frac{1}{\sqrt{\pi}} \int_{\mu}^{\infty} e^{-z^2/2} . dz.$$

Energy $(E_b) = \frac{A^2 T_b}{2}$, where $A =$ Carrier amplitude -

V Baseband pulse Transmission :-

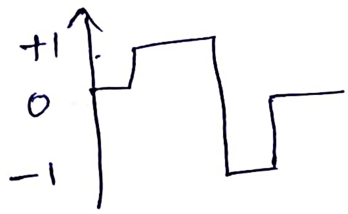
1. Intersymbol interference (ISI)
2. Background noise (AWGN).

1) Line coding :- (unipolar format)



Polar format :
Symbol 1 → +ve pulse
0 → -ve pulse.

Bipolar format :-



Symbol 0 – no pulse
1 → alternate +ve & -ve pulse.

Manchester format :-

1. Bi – ϕ – Mark.
2. Bi – ϕ – S (space)

1) M- ary Signaling :
→ non-binary symbols.

2) Properties of line coding:-

1. Transmission Bandwidth
2. Power efficiency
3. Error detection & correction
4. Power spectral density
5. Transparency.

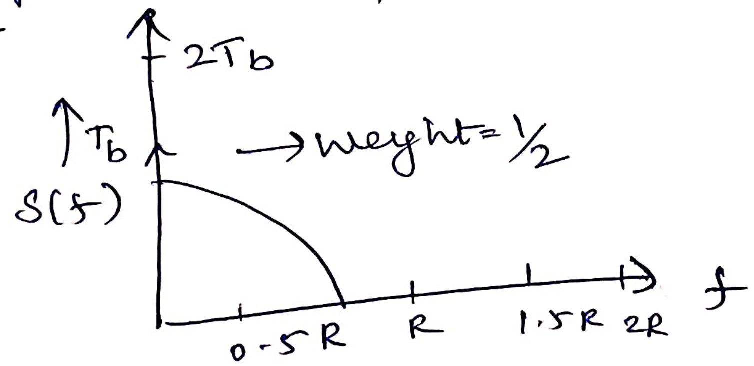
3) Power spectra of line codes:-

$$R_A(m) = E[a_n a_{n+m}] = \sum_{i=1}^{l} (a_n a_{n+m})_i \cdot P_i$$

1) Unipolar NRZ:

$$\boxed{R_A(0) = \frac{A^2}{2}}$$

$$S_{\substack{\text{unipolar}\\ NRZ}}(f) = \frac{A^2 T_b}{4} \operatorname{sinc}^2(\pi f T_b)\left[1 + \frac{1}{T_b}\delta(f)\right]$$



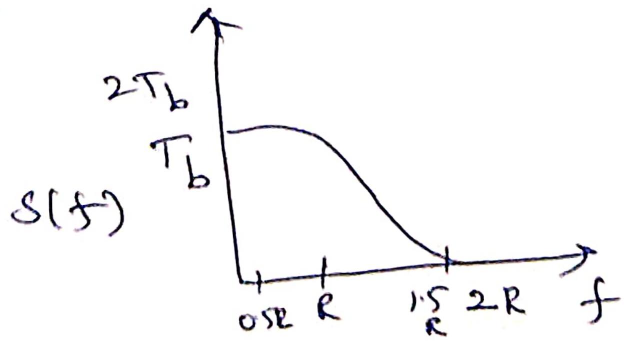
2) Polar NRZ

$$S_{\substack{\text{polar}\\ NRZ}}(f) = T_b \cdot A^2 \cdot \operatorname{sin}^2 c(\pi f T_b)$$

$$\boxed{P_n = A^2}$$

VI. normalized PSD of bipolar NRZ,

$$S_{\substack{\text{bipolar}\\ NRZ}}(f) = T_b \operatorname{sin} c^2(\pi f T_b)$$



NRZ-Bipolar (AMI)-Multibinary code :-

Power spectra of unipolar RZ.

$$S_{\text{unipolar RZ}}(f) = \frac{4T_b}{16} \cdot \text{sinc}^2\left(\frac{\pi f T_b}{2}\right)\left[1 + \frac{1}{T_b}\sum_{m=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\right]$$

5) Polar RZ

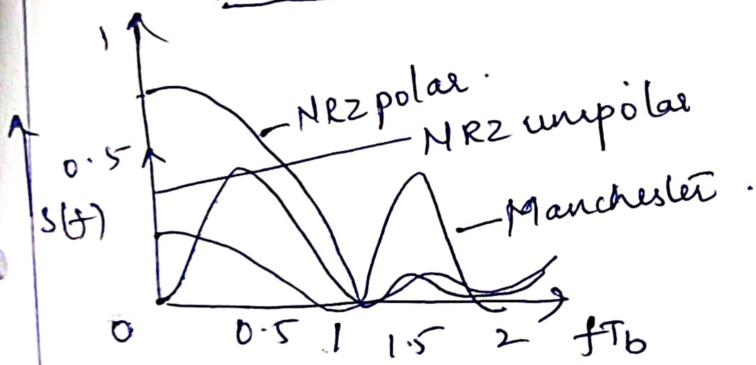
$$S_{\text{polar RZ}}(f) = \frac{A^2}{4} T_b \,\text{sinc}^2\left(\pi f \frac{T_b}{2}\right)$$

$$\boxed{S(f) = \frac{T_b}{2}\,\text{sinc}^2\left(\pi f \frac{T_b}{2}\right)}$$

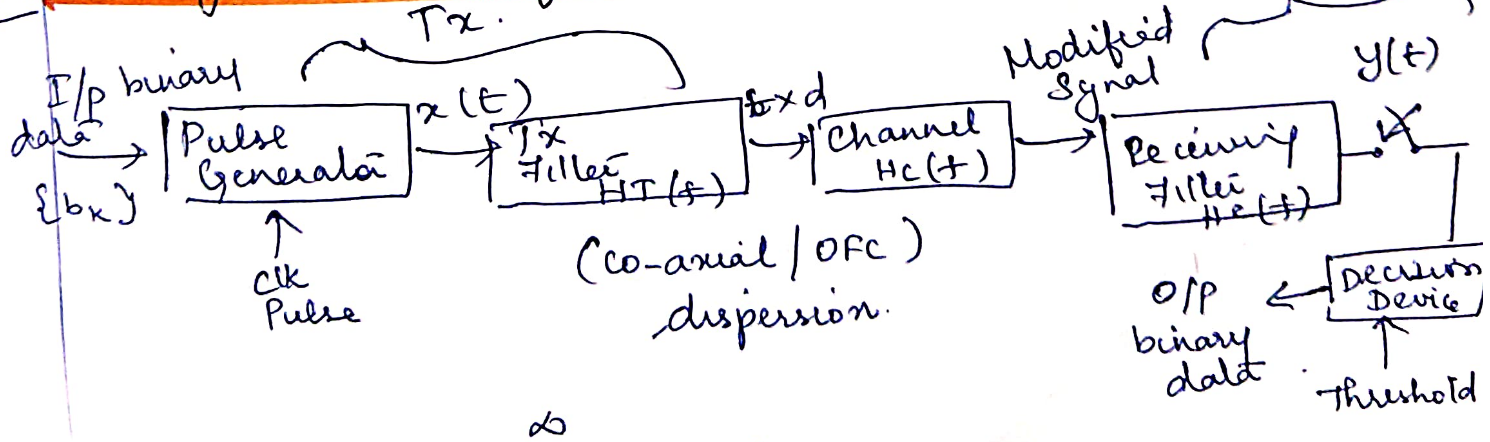
6) PSD of manchester encoding :-

$$F(f) = j T_b \,\text{sinc}\left(\pi f \frac{T_b}{2}\right) \sin\left(\frac{\omega T_b}{4}\right)$$

$$\boxed{S(f) = T_b \,\text{sinc}\left(\pi f \frac{T_b}{2}\right) \sin^2\left(\pi f \frac{T_b}{2}\right)}$$



VII Intersymbol Interference (ISI) :-

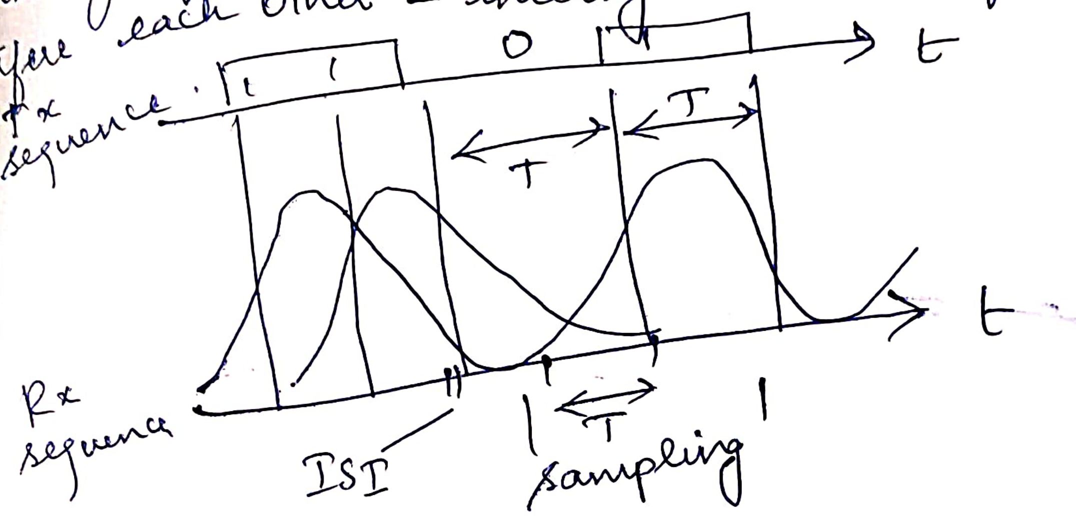


$$x(t) = \sum_{K=-\infty}^{\infty} a_K V(t - KT_b).$$

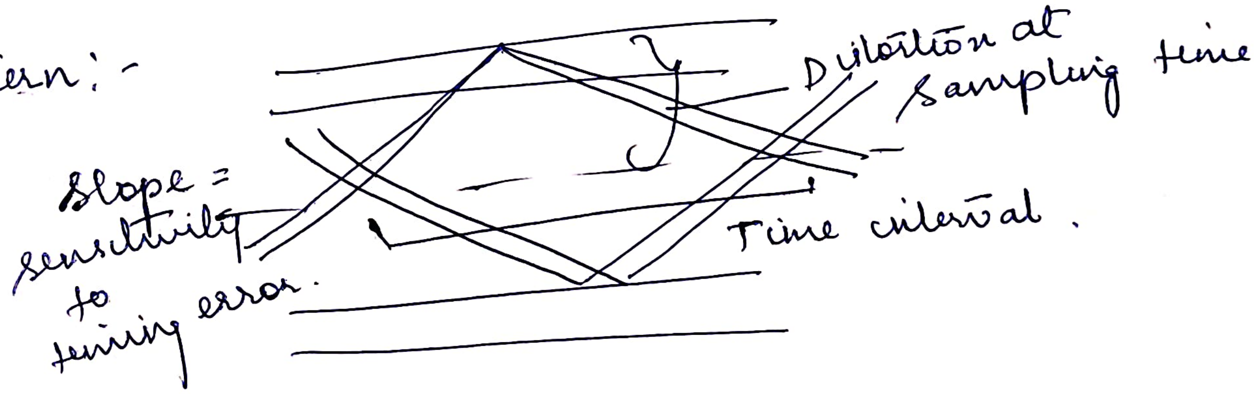
$$y(i) = \mu \sum_{k=-\infty} a_k p(t - kT_b)$$

$$\therefore \boxed{y(t_T) = \mu a_i + \mu \sum_{\substack{k \neq i \\ k=-\infty}}^{\infty} a_k P(iT_b - kT_b)}$$

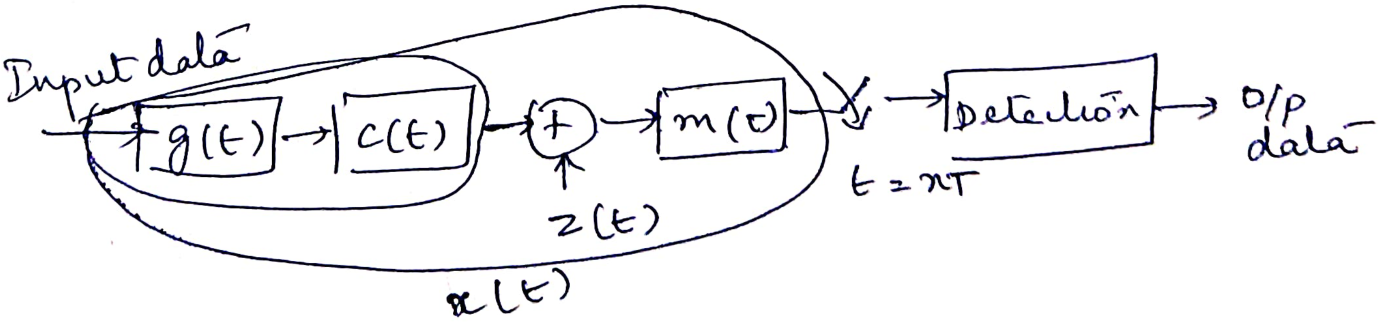
* Seq. of short pulses are txd thro' the system, one pulse every T_b seconds, the dispersed responses originating from differential symbol intervals will interfere each other – intersymbol interference.



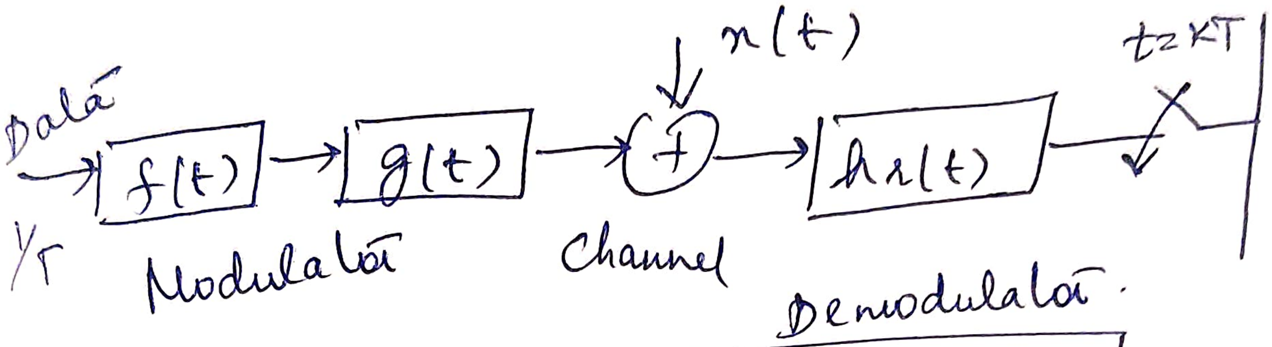
Eye pattern:-



Optimum Demodulation of Digital signal over Band limited channels:-



$$y(KT) = \int_{-\infty}^{\infty} r(t)\cdot h(t-KT)\,dt.$$



$$\boxed{y_K = \sum_{m=-N}^{N} h_m x_{m-K} + \eta_K}$$