

**EE6012/COMPUTER AIDED DESIGN OF ELECTRICAL APPARATUS  
UNIT-1 INTRODUCTION**

**1. Give the principle of energy conversion.**

Its state that the energy cannot be created or destroyed. it can only be converted from one form to the another form of energy. If we consider electric Motor then its convert electrical energy into mechanical energy.

**2. State the principle of electromechanical energy conversion.**

- Electromechanical energy conversion theory is the cornerstone for the analysis of electromechanical motion devices.
- It allows us to express the electromagnetic force or torque in terms of the device variables such as the currents and the displacement of the mechanical system.
- For energy conversion between electrical and mechanical forms, electromechanical devices are developed.

**3. List the various computational tools available for the design of electrical apparatus.**

- CAD-CAM
- CAD of electrical machines and power equipments
- MATLAB

**4. Compare the conventional and computer aided design of electrical apparatus.**

CONVENTIONAL APPROACH	CAD DESIGN
Only 2D design is possible.	Two and three dimensional can be designed.
Storage space required is more.	Reduced storage space.
Errors can happen while designing.	Errorless design in electrical apparatus.

**5. List the significance of co energy.**

It is a non-physical quantity, measured in energy units, used in theoretical analysis of energy in physical systems. The concept of co-energy can be applied to many conservative systems (inertial mechanical, electromagnetic, etc.), which can be described by a linear relationship between the input and stored energy.

**6. Write the limitations of conventional approach.**

- Only 2D design is possible.
- Storage space required is more.
- Errors can happen while designing.

**7. List the major considerations for design of electrical machines.**

- Specification
- Choice of materials.
- Assumption of basic design parameters.
- Design process.
- Performance calculation.
- Comparison.

**8. Write the various method of design.**

- Analysis .
- Synthesis.
- Hybrid process.

**9. What are the optimization techniques in design of electrical machines.**

1. Random search method. 2. Hooks & Jeeves method 3. Simplex method.

**10. Compare electric fields and magnetic fields.**

ELECTRIC FIELD	MAGNETIC FIELD
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The area around a magnet within which magnetic force is exerted, is called a magnetic field. It is produced by moving electric charges. The presence and strength of a magnetic field is denoted by “magnetic flux lines”.

Electric fields are generated around particles that bear electric charge. Positive charges are drawn towards it, while negative charges are repelled.

## UNIT II MATHEMATICAL FORMULATION OF FIELD PROBLEMS

### 1. Evaluate the expression for Poisson’s and Laplace equation.

Poisson's Equation:  $\nabla^2 V = -\rho_v/x$

Laplace Equation:  $\nabla^2 V = 0$

### 2. Write the expression for energy stored in magnetic field.

$$W = \frac{1}{2} (LI^2)$$

where L is the Inductance, I is the Current

### 3.. Define the magnetic vector potential.

It is defined as that quantity whose curl gives the magnetic flux density

$$\mathbf{B} = \nabla \times \mathbf{A}$$

where A is the magnetic vector potential

$$\mathbf{A} = \frac{\mu}{4\pi} \iiint \frac{\mathbf{J}}{r} dr$$

It is used to find the magnetic field due to current J.

### 4. Define electric flux density.

Electric flux density or displacement density is defined as the electric flux per unit area.

$$D = \frac{Q}{A} \text{ Coulomb/m}^2$$

For a sphere surface area,  $A = 4\pi r^2$

$$D = \frac{Q}{4\pi r^2}$$

But  $E = Q/4\pi\epsilon_0 r^2$  then  $D = \epsilon_0 E$

### 5. Define electric potential and give the expression.

Potential at any point is defined as the work done in moving a unit positive charge from infinity to that point in an electric field.

$$V = Q/4\pi\epsilon_0 r \text{ volts}$$

### 6. Give four similarities between electrostatic field and magnetic field.

ELECTROSTATIC FIELD	MAGNETIC FIELD
Electric field intensity E volts/m	Magnetic field intensity H A/m
Electric flux density $D = \epsilon_0 E$ C/m <sup>2</sup>	Magnetic flux intensity $B = \mu H$ w/m <sup>2</sup>
Energy stored is $\frac{1}{2} C V^2$ J	Energy stored is $\frac{1}{2} LI^2$ J
Charges are at rest	Charges are in motion

### 7. Define self inductance.

The self induction of a coil is defined as the ratio of total magnetic flux linkage with the circuit to the current through the coil.  $L = \Phi/i$  where  $\Phi$  is magnetic flux,  $i$  is the current

### 8. Give the expression for torque equation on closed circuits.

The torque on closed circuit in a magnetic field is

$$T = B I A \sin \Theta$$

where B is the magnetic vector potential.

### **9. Define vector and scalar field.**

- A scalar field is specified by a single number at each point.
- A vector field is specified by both magnitude and direction at each point in space.

### **10. State Gauss law.**

The electric flux passing through any closed surface is equal to the total charge enclosed on the surface.  $\chi = Q$

## **UNIT III PHILOSOPHY OF FEM**

### **1. Define finite difference method.**

The principle of finite difference methods is close to the numerical schemes used to solve ordinary differential equations. It consists in approximating the differential operator by replacing the derivatives in the equation using differential quotients. The domain is partitioned in space and in time and approximations of the solution are computed at the space or time points.

### **2. List out the general steps of finite difference method.**

In applying the method of finite differences a problem is defined by:

- A partial differential equation such as Poisson's equation
- A solution region
- Boundary and/or initial conditions.

An FDM method divides the solution domain into finite discrete points and replaces the partial differential equations with a set of difference equations. Thus the solutions obtained by FDM are not exact but approximate.

### **3. Mention few analytical methods for field computation.**

- Time varying field in conductors.
- Charge relaxation
- Propagation of EM waves

### **4. Define finite element method.**

The finite element method has its origin in the field of structural analysis. The finite elements analysis of any problem involves basically four steps:

- (A) Discretizing the solution region into a finite number of sub regions or elements ,
- (B) Deriving governing equations for a typical element ,
- (C) Assembling all the elements in the solution region, and
- (D) Solving the system of equations obtained.

### **5. What is shape function?**

The shape function is the function which interpolates the solution between the discrete values obtained at the mesh nodes. Therefore, appropriate functions have to be used and, as already mentioned, low order polynomials are typically chosen as shape functions. In this work linear shape functions are used.

### **6. What are the different solution techniques?**

- Finite element method
- Finite difference method

### **7. What are the significance of finite element analysis to an electromagnetic field computation problems?**

FEM is used in solving electromagnetic, magneto static, and quasi static problems.it is based on the variational principle for solving any boundary value field problems.

### **8.What do you mean by stiffness matrix?**

In the finite element method for the numerical solution of elliptic partial differential equations, the stiffness matrix represents the system of linear equations that must be solved in order to ascertain an approximate solution to the differential equation. ... In FEM, these are written in matrix form.

### **9. What are the advantages of FEM over FDM?**

FEM has gained popularity in solving electromagnetic, magnetostatic, and quasi-static magnetic field problems, the FDM has been in significant use for solving electrostatic problems, because in electrostatics, there are quite larger range of problems with open boundaries.

### **10. What do you mean by Discretisation?**

Discretization with the Finite Element Method. PARTIAL DIFFERENTIAL EQUATIONS (PDEs) are widely used to describe and model physical phenomena in different engineering fields and so also in microelectronics' fabrication.The most universal numerical method is based on finite elements.

## **UNIT IV CAD PACKAGES**

### **1. List the advantages of CAD.**

- Decrease in error percentage.
- Improved accuracy.
- Perform complex design analysis in short time.

### **2. Define preprocessing.**

The Preprocessing involves the preparations of data, such as nodal coordinates, connectivity, boundary conditions and loading and material information. The preparation of data require considerable effort if all data are to be handled manually. If the model is small, the user can often just write a text file and feed it into the processor, but as the complexity of the model grows and the number of elements increase, writing the data manually can be very time consuming and error-prone. Its therefore necessary with a computer preprocessor which help with mesh plotting and boundary conditions plotting.

### **3.List the steps of preprocessing.**

- 1.The first step in meshing would be to select a suitable element to mesh your domain.
- 2.To select a 2 Dimensional mesh of 1st order (corner nodes only) mixed mesh containing rectangles and triangles.
3. Then define an element as the element type that you want to fill the nodes with (say trias) as follows

Example with trias: <element no>, <connecting node 1>, <connecting node 2>, <connecting node 3> 3.2

### **4. Explain the statement of the two dimensional field problem.**

In 2D field problems, the considered domain is a surface S and its boundary is a curve. Let  $\phi$  be the unknown function that is to be determined. It is a scalar function of the space co-ordinates x and y. i.e.,  $\phi = \phi(x,y)$ .the time dependence is omitted.let F be the forcing equation which is also function of x and y, independent of time. The 2D field problem is defined by the differential second order is given by,

$$-\text{div}(\alpha \cdot \text{grad}\phi) + \beta\phi = -\frac{\partial}{\partial x}\left(\alpha_x \frac{\partial\phi}{\partial x}\right) - \frac{\partial}{\partial y}\left(\alpha_y \frac{\partial\phi}{\partial y}\right) + \beta\phi = f$$

### **5. Prepare the list of different boundary condition.**

1. Dirichlet's condition
2. Neumann's condition.
3. Cauchy boundary condition

### **6. Define mesh generation.**

Mesh generation is the practice of generating a polygonal or polyhedral mesh that approximates a geometric domain. The term "grid generation" is often used interchangeably. Typical uses are for rendering to a computer screen or for physical simulation such as finite element analysis or computational fluid dynamics.

**7. Discuss the flux normal boundary condition.**

The Neumann (or second-type) boundary condition is a type of boundary condition, named after Carl Neumann.<sup>[1]</sup> When imposed on an ordinary or a partial differential equation, the condition specifies the values in which the derivative of a solution is applied within the boundary of the domain. It is possible to describe the problem using other boundary conditions: a Dirichlet boundary condition specifies the values of the solution itself (as opposed to its derivative) on the boundary, whereas the Cauchy boundary condition, mixed boundary condition and Robin boundary condition are all different types of combinations of the Neumann and Dirichlet boundary conditions.

**8. List the steps involved in designing a machine using computer aided design.**

- Input data is to be fed into the program
- Applicable constraints/maximum or minimum permissible limits.
- Output data to be printed after execution of the program.

**9. Compare different boundary conditions.**

<b>Dirichlet's boundary condition</b>	<b>Neumann's boundary condition</b>
When imposed on an ordinary or a partial differential equation, it specifies the values that a solution needs to take on along the <b>boundary</b> of the domain.	When imposed on an ordinary or a partial differential equation, the condition specifies the values in which the derivative of a solution is applied within the boundary of the domain
Eg. In <u>electrostatics</u> , where a node of a circuit is held at a fixed voltage.	Eg. A perfect insulator would have no flux while an electrical component may be dissipating at a known power.

**10. What is post-processing?**

The postprocessing stage deals with the representation of results. Typically, the deformed configuration, mode shapes, temperature, and stress distribution are computed and displayed at this stage.

## UNIT V DESIGN APPLICATIONS

### 1. What are the different types of insulators?

1. Pin Insulator
2. Suspension Insulator
3. Strain Insulator

### 2. Write the significance of capacitance of a conductor.

Capacitance of a conductor depends on the size, shape and surroundings of the conductor. In case of isolated conductor, when the charge on the conductor is gradually increased, then its potential will also be increased gradually. Therefore, the charge on the conductor is proportional to the potential.

### 3. Give the equation of inductance of a solenoid.

$$L = \frac{\mu_0 N^2 A}{l}$$

N=Number of turns a=Area of cross section l=length of solenoid  $\mu_0$ =Free space permeability

### 4. Define mutual capacitance.

Mutual capacitance is intentional or unintentional capacitance that occurs between two charge-holding objects or conductors, in which the current passing through one passes over into the other.

### 5. Give the expression of torque for switched reluctance motor.

$$\text{Torque } T = \frac{1}{2} I^2 \frac{dL}{dl}$$

Where I is the current in amps, L is the length of the coil.

### 6. List the advantages of suspension type insulator.

It has the cheaper price, high strength, easy to install than pin type insulator for voltages beyond 36KV.

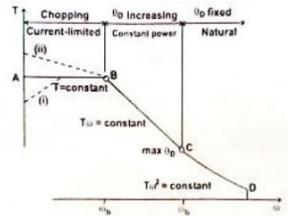
### 7. What is solenoid actuator?

The solenoid actuators work on the same basic principle as the electromechanical relay seen in the previous tutorial and just like relays, they can also be switched and controlled using transistors or MOSFET's. A "Linear Solenoid" is an electromagnetic device that converts electrical energy into a mechanical pushing or pulling force or motion.

### 8. Write the boundary conditions of magnetic field.

a) flux tangential condition b) flux normal condition c) Periodic boundary condition

### 9. Draw the torque speed capability curve.



Torque speed characteristic of switched reluctance motor

### 10. Write the torque equation of SRM.

$$\text{Torque } T = \frac{1}{2} i^2 \frac{dL}{d}$$

## UNIT I INTRODUCTION

### 1. Describe in detail the step by step conventional machine design procedure.

#### Specific magnetic loading:

Following are the factors which influences the performance of the machine.

##### (i) Iron loss:

A high value of flux density in the air gap leads to higher value of flux in the iron parts of the machine which results in increased iron losses and reduced efficiency.

##### (ii) Voltage:

When the machine is designed for higher voltage space occupied by the insulation becomes more thus making the teeth smaller and hence higher flux density in teeth and core.

##### (iii) Transient short circuit current:

A high value of gap density results in decrease in leakage reactance and hence increased value of armature current under short circuit conditions.

##### (iv) Stability:

The maximum power output of a machine under steady state condition is indirectly proportional to synchronous reactance. If higher value of flux density is used it leads to smaller number of turns per phase in armature winding. This results in reduced value of leakage reactance and hence increased value of power and hence increased steady state stability.

##### (v) Parallel operation:

The satisfactory parallel operation of synchronous generators depends on the synchronizing power. Higher the synchronizing power higher will be the ability of the machine to operate in synchronism. The synchronizing power is inversely proportional to the synchronous reactance and hence the machines designed with higher value air gap flux density will have better ability to operate in parallel with other machines.

#### Specific Electric Loading:

##### (i) Copper loss:

Higher the value of  $q$  larger will be the number of armature of conductors which results in higher copper loss. This will result in higher temperature rise and reduction in efficiency.

##### (ii) Voltage:

A higher value of  $q$  can be used for low voltage machines since the space required for the insulation will be smaller.

##### (iii) Synchronous reactance:

High value of  $q$  leads to higher value of leakage reactance and armature reaction and hence higher value of synchronous reactance. Such machines will have poor voltage regulation, lower value of current under short circuit condition and low value of steady state stability limit and small value of synchronizing power.

##### (iv) Stray load losses:

With increase of  $q$  stray load losses will increase.

## **2.Explain the different approaches of computer aided design of electrical apparatus.**

The process of design by CAD follows:

1. Analysis      2.Synthesis    3.Hybrid Process

### **ANALYSIS:**

In this process the dimensions of the machine are estimated by experience selecting suitable volume making use of output equation and thus estimating all the dimensions of the m/c and the performance by known methods. The performance so estimated is compared with the desired result as specified and any divergence is eliminated by successive iterations by making small changes in dimensions. Here computer is used as a calculating aid.

### **SYNTHESIS:**

The process of synthesis is the exact opposite of the Analysis. Here the starting point is the desired performance and the computer is required to work backward and determine the optimum machine dimensions. The process involves the formulation of suitable inverted performance equations which are differential equations connecting the performance to the various design parameters like length, diameter, air-gap, current density etc. The designer is also required to feed in the boundary conditions or constraints of the equations. This method makes full use of the logic abilities of the computer and theoretically the most desirable method for design using computer.

### **HYBRID PROCESS:**

It is a combination of the Analysis & Synthesis and involves partial synthesis using the standard frames, slots & conductors decided on the basis of availability in the market. It is a practical method because it makes possible the use of standardization which is important for economic and practical design. Since the synthesis methods involve greater cost, the major part of the program is based upon analysis with a limited portion of the program being based upon synthesis. This approach makes the design more practical and economical.

### **3. Discuss in detail the limitation of conventional machine design over the computer aided design.**

#### **SATURATION:**

In the designing of the electromagnetic machines we use ferromagnetic material. The flux density of the machine is determined by the saturation of the ferromagnetic material used. Higher flux density results in higher cost.

#### **TEMPERATURE RISE:**

The most important part of the machine is the insulation. It should be according to the maximum temperature in the machine. If the operating temperature is higher than the allowable temperature its life will be drastically reduced. Proper ventilation techniques are used to keep the temperature rise in the safer limits. The coolant will allow the heat from the machine to dissipate.

#### **INSULATION:**

The insulation is the most important part as it should withstand the electrical mechanical and thermal stress produced by the machine. Transformers are the machines which should have higher insulation where the large axial and radial forces are produced when the secondary winding of the transformer is short-circuited with primary on. It should withstand high mechanical stress due to secondary winding is short circuited.

#### **EFFICIENCY:**

Efficiency should be high to reduce the operation cost of the machine. So it requires large amount of materials to design. We can reduce the operating cost by increasing the design cost.

#### **MECHANICAL PARTS:**

The construction of the mechanical parts should be economical but it should satisfy the requirements of performance, reliability, and durability. For the high speed machines it is very important because it will be having more mechanical stress at the rotor. The length of the air gap is reduced to increase the high power factor.

#### **COMMUTATION:**

This problem only occurs in the commutator machines. As it decreases the maximum output taken from the machine.

#### **POWER FACTOR:**

Poor power factor results in large amount of current in the same power, therefore large conductor sizes have to be used. It mostly affects the induction motor.

#### **CONSUMER SPECIFICATION:**

The important in this is it should satisfy the consumer specification with their economic constrains. The design should evolve in this manner.

**STANDARD SPECIFICATION:**

This specification cannot be neglected by both consumer and manufacturer.

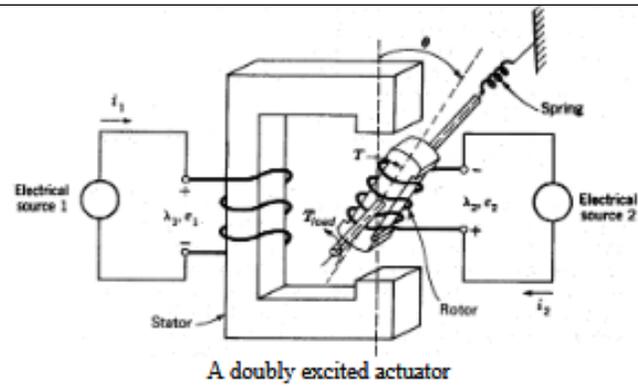
**4. With neat sketch explain the multiple excited magnetic field systems in electromechanical energy conversion system. Also calculate the expression for field energy in the system.**

The general principle for force and torque calculation discussed above is equally applicable to multi-excited systems. Consider a doubly excited rotating actuator shown schematically in the diagram below as an example. The differential energy and coenergy functions can be derived as following:

$$dW_f = dW_e - dW_m$$

Where,

$$dW_e = e_1 i_1 dt + e_2 i_2 dt$$



$$e_1 = \frac{d\lambda_1}{dt}, \quad e_2 = \frac{d\lambda_2}{dt}$$

and

$$dW_m = Td\theta$$

Hence,

$$\begin{aligned} dW_f(\lambda_1, \lambda_2, \theta) &= i_1 d\lambda_1 + i_2 d\lambda_2 - Td\theta \\ &= \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \lambda_1} d\lambda_1 + \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \lambda_2} d\lambda_2 \\ &\quad + \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta} d\theta \end{aligned}$$

and

$$\begin{aligned} dW_f'(i_1, i_2, \theta) &= d[i_1 \lambda_1 + i_2 \lambda_2 - W_f(\lambda_1, \lambda_2, \theta)] \\ &= \lambda_1 di_1 + \lambda_2 di_2 + Td\theta \\ &= \frac{\partial W_f'(i_1, i_2, \theta)}{\partial i_1} di_1 + \frac{\partial W_f'(i_1, i_2, \theta)}{\partial i_2} di_2 \\ &\quad + \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta} d\theta \end{aligned}$$

Therefore, comparing the corresponding differential terms, we obtain

$$T = - \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta}$$

or

$$T = \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta}$$

For magnetically linear systems, currents and flux linkages can be related by constant inductances as following

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

or

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

where  $L_{12}=L_{21}$ ,  $\Gamma_{11}=L_{22}/\Delta$ ,  $\Gamma_{12}=\Gamma_{21}=-L_{12}/\Delta$ ,  $\Gamma_{22}=L_{11}/\Delta$ , and  $\Delta=L_{11}L_{22}-L_{12}^2$ . The magnetic energy and coenergy can then be expressed as

$$W_f(\lambda_1, \lambda_2, \theta) = \frac{1}{2} \Gamma_{11} \lambda_1^2 + \frac{1}{2} \Gamma_{22} \lambda_2^2 + \Gamma_{12} \lambda_1 \lambda_2$$

and

$$W_f'(i_1, i_2, \theta) = \frac{1}{2} L_{11} i_1^2 + \frac{1}{2} L_{22} i_2^2 + L_{12} i_1 i_2$$

respectively, and it can be shown that they are equal.

Therefore, the torque acting on the rotor can be calculated as

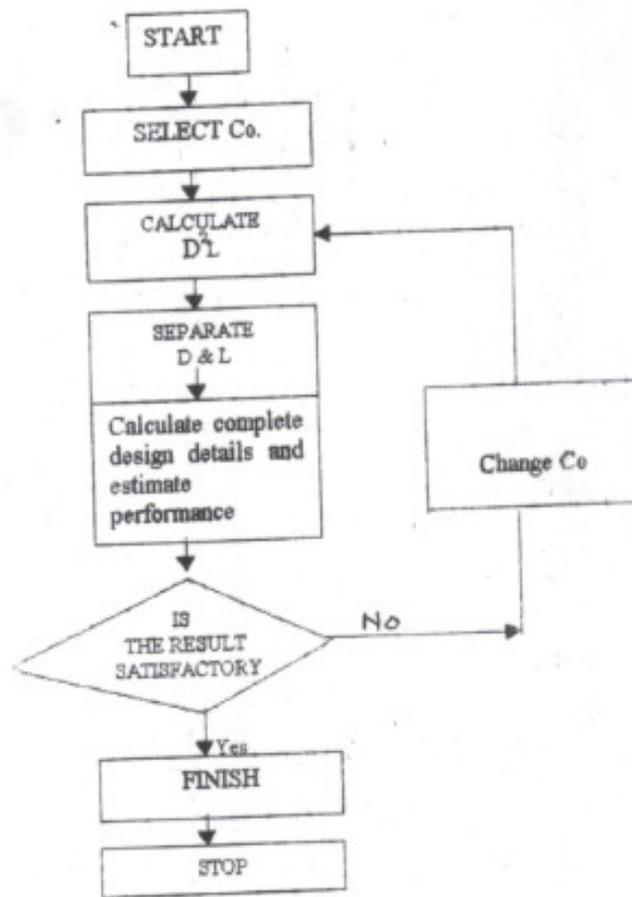
$$\begin{aligned} T &= -\frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta} = \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta} \\ &= \frac{1}{2} i_1^2 \frac{dL_{11}(\theta)}{d\theta} + \frac{1}{2} i_2^2 \frac{dL_{22}(\theta)}{d\theta} + i_1 i_2 \frac{dL_{12}(\theta)}{d\theta} \end{aligned}$$

Because of the salient (not round) structure of the rotor, the self inductance of the stator is a function of the rotor position and the first term on the right hand side of the above torque expression is nonzero for that  $dL_{11}/d\theta \neq 0$ . Similarly, the second term on the right hand side of the above torque expression is nonzero because of the salient structure of the stator. Therefore, these two terms are known as the reluctance torque component. The last term in the torque expression, however, is only related to the relative position of the stator and rotor and is independent of the shape of the stator and rotor poles.

## 5. Describe in detail the analysis and synthesis method of Design with flow chart.

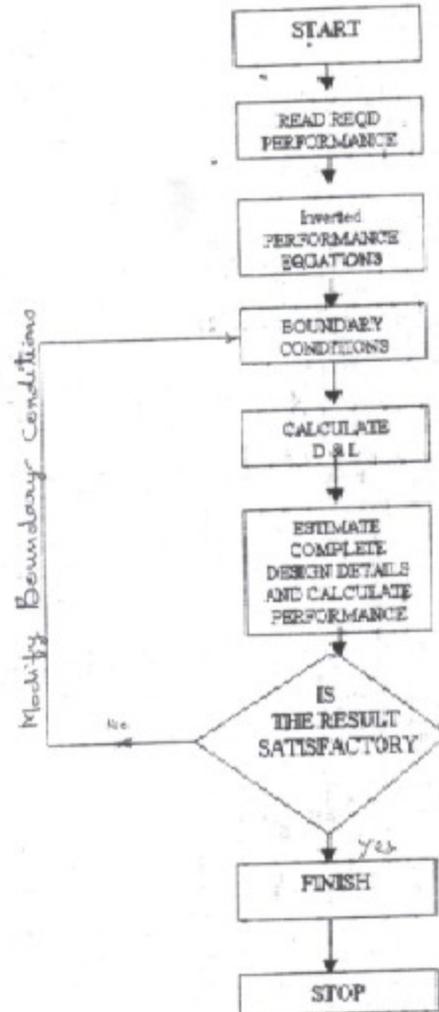
### ANALYSIS:

In this process the dimensions of the machine are estimated by experience selecting suitable volume making use of output equation and thus estimating all the dimensions of the m/c and the performance by known methods. The performance so estimated is compared with the desired result as specified and any divergence is eliminated by successive iterations by making small changes in dimensions. Here computer is used as a calculating aid.



### SYNTHESIS:

The process of synthesis is the exact opposite of the Analysis. Here the starting point is the desired performance and the computer is required to work backward and determine the optimum machine dimensions. The process involves the formulation of suitable inverted performance equations which are differential equations connecting the performance to the various design parameters like length, diameter, air-gap, current density etc. The designer is also required to feed in the boundary conditions or constraints of the equations. This method makes full use of the logic abilities of the computer and theoretically the most desirable method for design using computer.



## 6. Discuss in detail the about the design procedure for computer aided design.

### GIVEN SPECIFICATION:

Consists of performance requirement as defined by the customer's need and specifications.

### CHOICE OF MATERIALS ETC:

Based on given specifications, the designer choses materials : magnetic, conducting and insulating for electrical design and other materials for frame, bearings etc. This depends upon the availability of materials & manufacturers specifications.

### ASSUMPTIONS OF BASIC DESIGN PARAMETERS:

Such specific magnetic & electric loading, space factor, stacking factor, etc.

### DESIGN PROCESS:

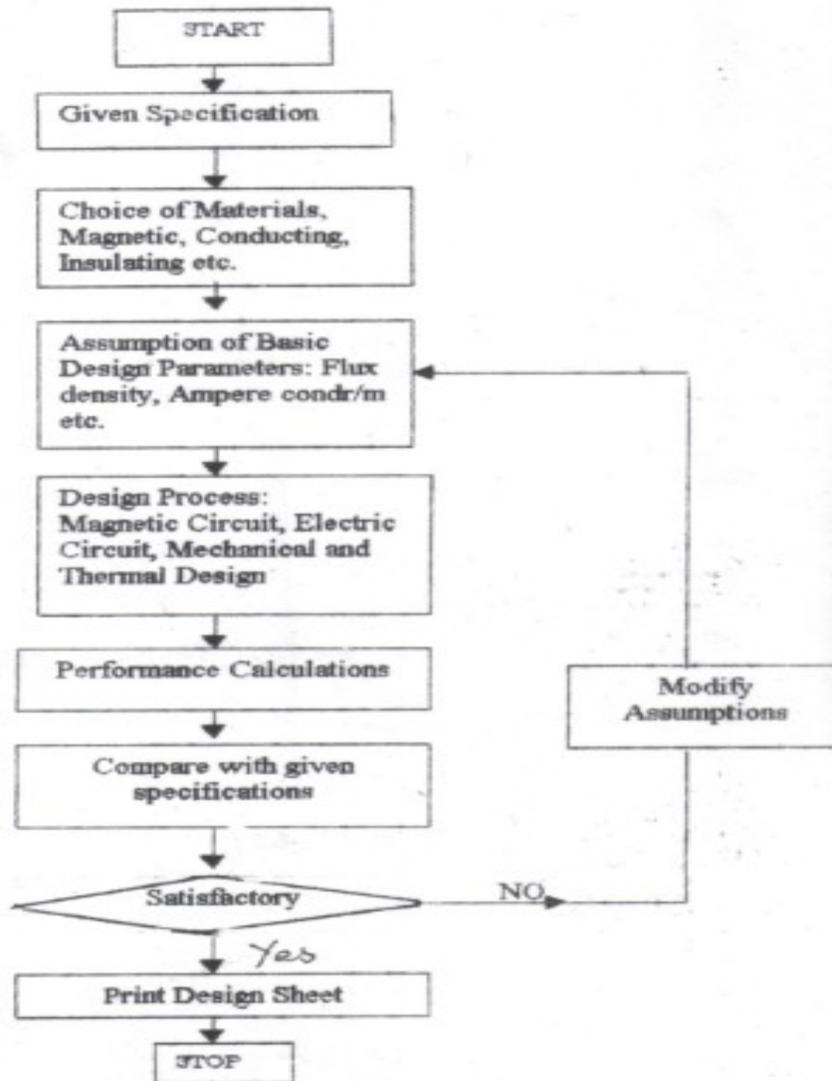
Consists of calculations to estimate the various dimensions of magnetic and electric circuits and making thermal and mechanical designs.

### PERFORMANCE CALCULATION:

Predetermination of performance under no load & load conditions, estimation of temp. rise efficiency, regulation & cost etc.

### COMPARISON:

Compare the estimated performance with the customer's requirement. If not satisfactory (which is generally the case), modify the basic assumptions so as to bring the final design closer to the objective.



### 7. Describe in detail the principle of energy conversion in an electromagnetic field.

For energy conversion between electrical and mechanical forms, electromechanical devices are developed. In general, electromechanical energy conversion devices can be divided into three categories: (1) Transducers (for measurement and control)

These devices transform the signals of different forms. Examples are microphones,

pickups, and speakers.

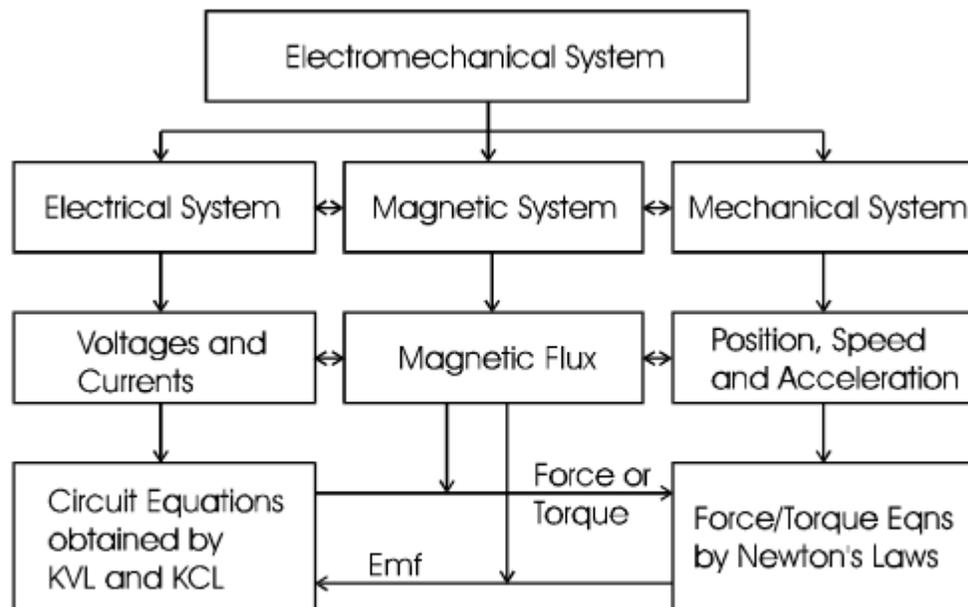
(2) Force producing devices (linear motion devices)

These type of devices produce forces mostly for linear motion drives, such as relays, solenoids (linear actuators), and electromagnets.

(3) Continuous energy conversion equipment

These devices operate in rotating mode. A device would be known as a generator if it convert mechanical energy into electrical energy, or as a motor if it does the otherway around (from electrical to mechanical). Since the permeability of ferromagnetic materials are much larger than the Permittivity of dielectric materials, it is more advantageous to use electromagnetic field as the medium for electromechanical energy conversion. As illustrated in the following diagram, an electromechanical system consists of an electrical subsystem (electric circuits such as windings), a magnetic subsystem (magnetic field in the magnetic cores and airgaps), and a mechanical subsystem (mechanically movable parts such as a plunger in a linear actuator and a rotor in a rotating electrical machine). Voltages and currents are used to describe the state of the electrical subsystem and they are governed by the basic circuital laws: Ohm's law, KCL and KVL.

The state of the mechanical subsystem can be described in terms of positions, velocities, and accelerations, and is governed by the Newton's laws. The magnetic subsystem or magnetic field fits between the electrical and mechanical subsystems and acting as a "ferry" in energy transform and conversion. The field quantities such as magnetic flux, flux density, and field strength, are governed by the Maxwell's equations. When coupled with an electric circuit, the magnetic flux interacting with the current in the circuit would produce a force or torque on a mechanically movable part. On the other hand, the movement of the moving part will could variation of the magnetic flux linking the electric circuit and induce an electromotive force ( emf ) in the circuit. The product of the torque and speed (the mechanical power) equals the active component of the product of the emf and current. Therefore, the electrical energy and the mechanical energy are inter-converted via the magnetic field.



**Concept map of electromechanical system modeling**

The general concept of electromechanical system modeling will also be illustrated by a singly excited rotating system. Induced emf in Electromechanical Systems. The diagram below shows a conductor of

length  $l$  placed in a uniform magnetic field of flux density  $B$ . When the conductor moves at a speed  $v$ , the induced emf in the conductor can be determined by  $e = Lv \cdot B$

The direction of the emf can be determined by the "right hand rule" for cross products. In a coil of  $N$  turns, the induced emf can be calculated by

$$e = -d\lambda/dt$$

where  $\lambda$  is the flux linkage of the coil and the minus sign indicates that the induced current opposes the variation of the field. It makes no difference whether the variation of the flux linkage is a result of the field variation or coil movement. In practice, it would be convenient if we treat the emf as a voltage. The above expression can then be rewritten as

$$e = d\lambda/dt = L(di/dt) + i(dL/dx + dx/dt)$$

if the system is magnetically linear, i.e. the self inductance is independent of the current. It should be noted that the self inductance is a function of the displacement  $x$  since there is a moving part in the system.

## Force and Torque on a Current Carrying Conductor

The force on a moving particle of electric charge  $q$  in a magnetic field is given by the Lorentz's force law:

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

The force acting on a current carrying conductor can be directly derived from the equation as

$$\mathbf{F} = I \int_C d\mathbf{l} \times \mathbf{B}$$

where  $C$  is the contour of the conductor. For a homogeneous conductor of length  $l$  carrying current  $I$  in a uniform magnetic field, the above expression can be reduced to

$$\mathbf{F} = I(\mathbf{l} \times \mathbf{B})$$

In a rotating system, the torque about an axis can be calculated by

$$\mathbf{T} = \mathbf{r} \times \mathbf{F}$$

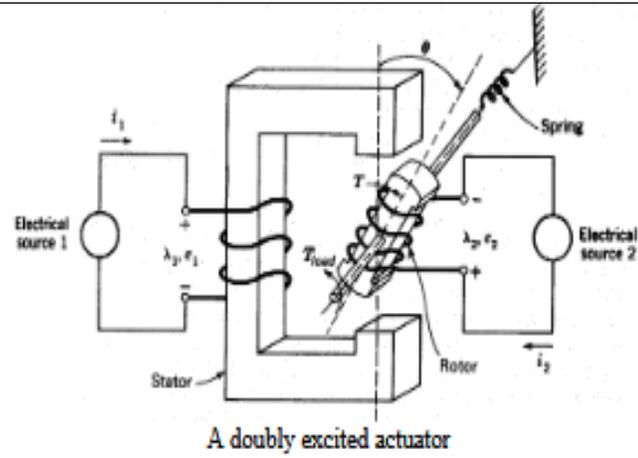
where  $\mathbf{r}$  is the radius vector from the axis towards the conductor.

The general principle for force and torque calculation discussed above is equally applicable to multi-excited systems. Consider a doubly excited rotating actuator shown schematically in the diagram below as an example. The differential energy and coenergy functions can be derived as following:

$$dW_f = dW_e - dW_m$$

Where,

$$dW_e = e_1 i_1 dt + e_2 i_2 dt$$



$$e_1 = \frac{d\lambda_1}{dt}, \quad e_2 = \frac{d\lambda_2}{dt}$$

and

$$dW_m = Td\theta$$

Hence,

$$\begin{aligned} dW_f(\lambda_1, \lambda_2, \theta) &= i_1 d\lambda_1 + i_2 d\lambda_2 - Td\theta \\ &= \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \lambda_1} d\lambda_1 + \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \lambda_2} d\lambda_2 \\ &\quad + \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta} d\theta \end{aligned}$$

and

$$\begin{aligned} dW_f'(i_1, i_2, \theta) &= d[i_1 \lambda_1 + i_2 \lambda_2 - W_f(\lambda_1, \lambda_2, \theta)] \\ &= \lambda_1 di_1 + \lambda_2 di_2 + Td\theta \\ &= \frac{\partial W_f'(i_1, i_2, \theta)}{\partial i_1} di_1 + \frac{\partial W_f'(i_1, i_2, \theta)}{\partial i_2} di_2 \\ &\quad + \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta} d\theta \end{aligned}$$

Therefore, comparing the corresponding differential terms, we obtain

$$T = - \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta}$$

or

$$T = \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta}$$

For magnetically linear systems, currents and flux linkages can be related by constant inductances as following

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

or

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

where  $L_{12}=L_{21}$ ,  $\Gamma_{11}=L_{22}/\Delta$ ,  $\Gamma_{12}=\Gamma_{21}=-L_{12}/\Delta$ ,  $\Gamma_{22}=L_{11}/\Delta$ , and  $\Delta=L_{11}L_{22}-L_{12}^2$ . The magnetic energy and coenergy can then be expressed as

$$W_f(\lambda_1, \lambda_2, \theta) = \frac{1}{2}\Gamma_{11}\lambda_1^2 + \frac{1}{2}\Gamma_{22}\lambda_2^2 + \Gamma_{12}\lambda_1\lambda_2$$

and

$$W_f'(i_1, i_2, \theta) = \frac{1}{2}L_{11}i_1^2 + \frac{1}{2}L_{22}i_2^2 + L_{12}i_1i_2$$

respectively, and it can be shown that they are equal.

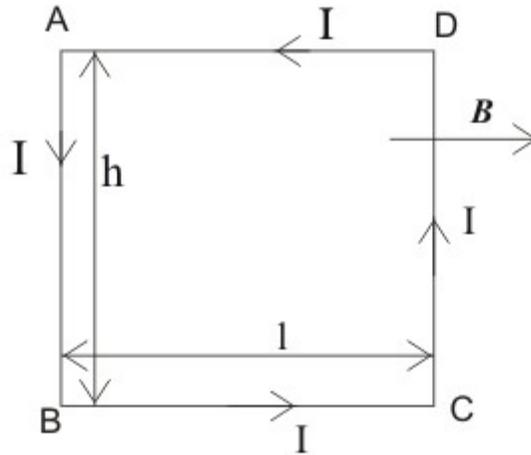
Therefore, the torque acting on the rotor can be calculated as

$$\begin{aligned} T &= -\frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta} = \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta} \\ &= \frac{1}{2}i_1^2 \frac{dL_{11}(\theta)}{d\theta} + \frac{1}{2}i_2^2 \frac{dL_{22}(\theta)}{d\theta} + i_1i_2 \frac{dL_{12}(\theta)}{d\theta} \end{aligned}$$

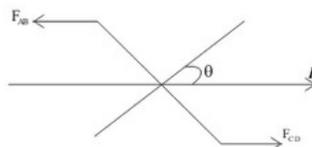
Because of the salient (not round) structure of the rotor, the self inductance of the stator is a function of the rotor position and the first term on the right hand side of the above torque expression is nonzero for that  $dL_{11}/d\theta \neq 0$ . Similarly, the second term on the right hand side of the above torque expression is nonzero because of the salient structure of the stator. Therefore, these two terms are known as the reluctance torque component. The last term in the torque expression, however, is only related to the relative position of the stator and rotor and is independent of the shape of the stator and rotor poles.

### 8. Derive expression for torque experienced by a differential current loop kept in a magnetic field.

Consider a rectangular loop ABCD being suspended in a uniform magnetic field B and direction of B is parallel to the plane of the coil as shown below in the figure



- Magnitude of force on side AB is  $F_{AB} = IhB$  (angle between  $I$  and  $B$  is  $90^\circ$ ) and direction of force as calculated from the right hand palm rule would be normal to the paper in the upwards direction
- Similarly magnitude of force on CD is  $F_{CD} = IhB$  and direction of  $F_{CD}$  is normal to the page but in the downwards direction going into the page
- The forces  $F_{AB}$  and  $F_{CD}$  are equal in magnitude and opposite in direction and hence they constitute a couple
- Torque  $\tau$  exerted by this couple on rectangular loop is  $\tau = IhlB$  Since torque = one of the force \* perpendicular distance between them
- No force acts on the side BC since current element makes an angle  $\theta = 0$  with  $B$  due to which the product  $(I \mathbf{L} \times \mathbf{B})$  becomes equal to zero
- Similarly on the side DA, no magnetic force acts since current element makes an angle  $\theta = 180^\circ$  with  $B$
- Thus total torque on rectangular current loop is  $\tau = IhlB = IAB$  Where  $A = hl$  is the area of the loop
- If the coil having  $N$  rectangular loop is placed in magnetic field then torque is given by  $\tau = NIAB$
- Again if the normal to the plane of coil makes an angle  $\theta$  with the uniform magnetic field as shown below in the figure then



$$\tau = NIAB \sin \theta$$

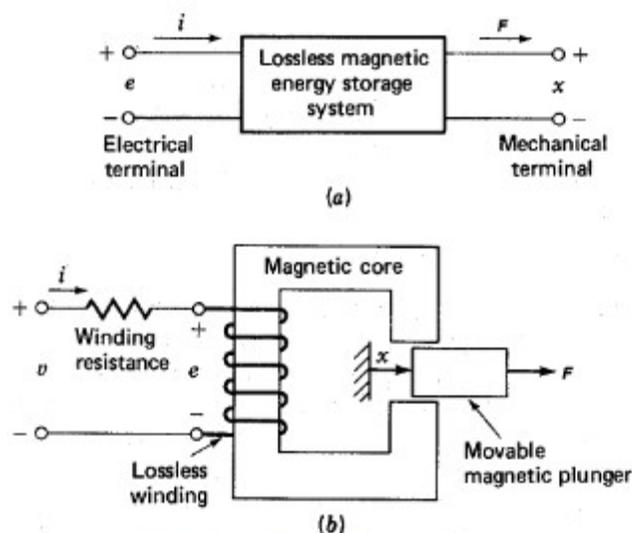
□ We know that when an electric dipole is placed in external electric field then torque experienced by the dipole is  $\tau = \mathbf{P} \times \mathbf{E} = PE \sin \theta$  Where  $\mathbf{P}$  is the electric dipole moment. Comparing expression for torque experienced by electric dipole with the expression for torque on a current loop i.e.,  $\tau = (NIA)B \sin \theta$  if we take  $NIA$  as magnetic dipole moment ( $\mathbf{m}$ ) analogous to electric dipole moment ( $\mathbf{p}$ ), we have  $\mathbf{m} = NIA$  then

$$\tau = \mathbf{m} \times \mathbf{B}$$

- The coil thus behaves as a magnetic dipole.
- The direction of magnetic dipole moment lies along the axis of the loop
- This torque tends to rotate the coil about its own axis. Its value changes with angle between the plane of the coil and the direction of the magnetic field
- Unit of magnetic moment is Ampere.meter<sup>2</sup> (Am<sup>2</sup>).

**9. With neat sketch explain the single excited magnetic field systems in electromechanical energy conversion system.**

Consider a singly excited linear actuator as shown below. The winding resistance is  $R$ . At a certain time instant  $t$ , we record that the terminal voltage applied to the excitation winding is  $v$ , the excitation winding current  $i$ , the position of the movable plunger  $x$ , and the force acting on the plunger  $F$  with the reference direction chosen in the positive direction of the  $x$ -axis, as shown in the diagram. After a time interval  $dt$ , we notice that the plunger has



moved for a distance  $dx$  under the action of the force  $F$ . The mechanical work done by the force acting on the plunger during this time interval is thus

$$dW = F dx$$

The amount of electrical energy that has been transferred into the magnetic field and converted into the mechanical work during this time interval can be calculated by subtracting the power loss dissipated in the winding resistance from the total power fed into the excitation winding as

$$dW_e = dW_f + dW_m = vidt - Ri^2 dt$$

Because

$$e = \frac{d\lambda}{dt} = v - Ri$$

we can write

$$dW_f = dW_e - dW_m = eidt - Fdx$$
$$= id\lambda - Fdx$$

From the above equation, we know that the energy stored in the magnetic field is a function of the flux linkage of the excitation winding and the position of the plunger. Mathematically, we can also write

$$dW_f(\lambda, x) = \frac{\partial W_f(\lambda, x)}{\partial \lambda} d\lambda + \frac{\partial W_f(\lambda, x)}{\partial x} dx$$

Therefore, by comparing the above two equations, we conclude

$$i = \frac{\partial W_f(\lambda, x)}{\partial \lambda} \quad \text{and} \quad F = -\frac{\partial W_f(\lambda, x)}{\partial x}$$

From the knowledge of electromagnetics, the energy stored in a magnetic field can be expressed as

$$W_f(\lambda, x) = \int_0^\lambda i(\lambda, x) d\lambda$$

For a magnetically linear (with a constant permeability or a straight line magnetization curve such that the inductance of the coil is independent of the excitation current) system, the above expression becomes

$$W_f(\lambda, x) = \frac{1}{2} \frac{\lambda^2}{L(x)}$$

and the force acting on the plunger is then

$$F = -\frac{\partial W_f(\lambda, x)}{\partial x} = \frac{1}{2} \left[ \frac{\lambda}{L(x)} \right]^2 \frac{dL(x)}{dx} = \frac{1}{2} i^2 \frac{dL(x)}{dx}$$

In the diagram below, it is shown that the magnetic energy is equivalent to the area above the magnetization or  $\lambda$ - $i$  curve. Mathematically, if we define the area underneath the magnetization curve as the *coenergy* (which does not exist physically), i.e.

$$W_f'(i, x) = i\lambda - W_f(\lambda, x)$$

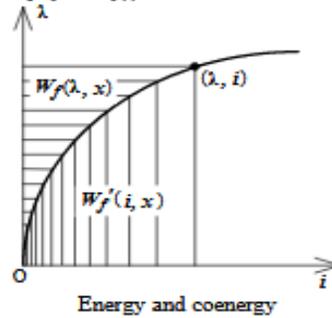
we can obtain

$$\begin{aligned} dW_f'(i, x) &= \lambda di + i d\lambda - dW_f(\lambda, x) \\ &= \lambda di + F dx \\ &= \frac{\partial W_f'(i, x)}{\partial i} di + \frac{\partial W_f'(i, x)}{\partial x} dx \end{aligned}$$

Therefore,

$$\lambda = \frac{\partial W_f'(i, x)}{\partial i}$$

and 
$$F = \frac{\partial W_f'(i, x)}{\partial x}$$



From the above diagram, the coenergy or the area underneath the magnetization curve can be calculated by

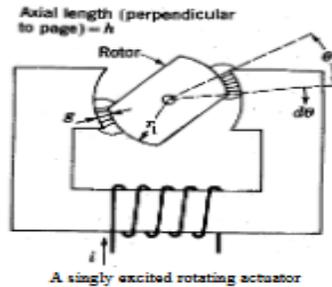
$$W_f'(i, x) = \int_0^i \lambda(i, x) di$$

For a magnetically linear system, the above expression becomes

$$W_f'(i, x) = \frac{1}{2} i^2 L(x)$$

and the force acting on the plunger is then

$$F = \frac{\partial W_f'(i, x)}{\partial x} = \frac{1}{2} i^2 \frac{dL(x)}{dx}$$



**Table: Torque in a singly excited rotating actuator**

Energy	Coenergy
<i>In general,</i>	
$dW_f = i d\lambda - T d\theta$	$dW_f' = \lambda di + T d\theta$
$W_f(\lambda, \theta) = \int_0^\lambda i(\lambda, \theta) d\lambda$	$W_f'(i, \theta) = \int_0^i \lambda(i, \theta) di$
$i = \frac{\partial W_f(\lambda, \theta)}{\partial \lambda}$	$\lambda = \frac{\partial W_f'(i, \theta)}{\partial i}$
$T = -\frac{\partial W_f(\lambda, \theta)}{\partial \theta}$	$T = \frac{\partial W_f'(i, \theta)}{\partial \theta}$
<i>If the permeability is a constant,</i>	
$W_f(\lambda, \theta) = \frac{1}{2} \frac{\lambda^2}{L(\theta)}$	$W_f'(i, \theta) = \frac{1}{2} i^2 L(\theta)$
$T = \frac{1}{2} \left[ \frac{\lambda}{L(\theta)} \right]^2 \frac{dL(\theta)}{d\theta} = \frac{1}{2} i^2 \frac{dL(\theta)}{d\theta}$	$T = \frac{1}{2} i^2 \frac{dL(\theta)}{d\theta}$

## 10. Describe about major considerations and limitations in electrical machine design.

### Major considerations in Electrical Machine Design

The basic components of all electromagnetic apparatus are the field and armature windings supported by dielectric or insulation, cooling system and mechanical parts. Therefore, the factors for consideration in the design are,

#### Magnetic circuit or the flux path:

Should establish required amount of flux using minimum MMF. The core losses should be less.

#### Electric circuit or windings:

Should ensure required EMF is induced with no complexity in winding arrangement.

The copper losses should be less.

**Insulation:**

Should ensure trouble free separation of machine parts operating at different potential and confine the current in the prescribed paths.

**Cooling system or ventilation:**

Should ensure that the machine operates at the specified temperature.

**Machine parts:**

Should be robust.

The art of successful design lies not only in resolving the conflict for space between iron, copper, insulation and coolant but also in optimization of cost of manufacturing, and operating and maintenance charges.

**The factors, apart from the above, that requires consideration are**

- a. Limitation in design (saturation, current density, insulation, temperature rise etc.)
- b. Customer's needs.
- c. National and international standards.
- d. Convenience in production line and transportation.
- e. Maintenance and repairs.

## UNIT-II MATHEMATICAL FORMULATIONS FOR FIELD PROBLEMS

### 1. Obtain the expression for the magnetic scalar potential.

In studying electric field problems, we introduced the concept of electric potential that simplified the computation of electric fields for certain types of problems. In the same manner let us relate the magnetic field intensity to a scalar magnetic potential and write:

$$\vec{H} = -\nabla V_m$$

From Ampere's law , we know that

$$\nabla \times \vec{H} = \vec{J}$$

Therefore

$$\nabla \times (-\nabla V_m) = \vec{J}$$

But using vector identity,  $\nabla \times (\nabla V) = 0$  we find that  $\vec{H} = -\nabla V_m$  is valid only where  $\vec{J} = 0$ . Thus

the scalar magnetic potential is defined only in the region where  $\vec{J} = 0$ . Moreover,  $V_m$  in general is not a single valued function of position.

## 2. Obtain the expression for the magnetic vector potential.

Electric fields generated by stationary charges obey,  $\nabla \times \mathbf{E} = \mathbf{0}$ , then  $\mathbf{E} = -\nabla\phi$ , since the curl of a gradient is automatically zero. In fact, whenever we come across an irrotational vector field in physics we can always write it as the gradient of some scalar field. This is clearly a useful thing to do, since it enables us to replace a vector field by a much simpler scalar field. The quantity  $\phi$  in the above equation is known as the electric scalar potential. Magnetic fields generated by steady currents (and unsteady currents, for that matter) satisfy.

$$\nabla \cdot \mathbf{B} = 0,$$

$$\text{Then, } \mathbf{B} = \nabla \times \mathbf{A}$$

Since the divergence of a curl is automatically zero. In fact, whenever we come across a solenoidal vector field in physics we can always write it as the curl of some other vector field. This is not an obviously useful thing to do, however, since it only allows us to replace one vector field by another. The quantity  $\mathbf{A}$  is known as the magnetic vector potential. We know from Helmholtz's theorem that a vector field is fully specified by its divergence and its curl. The curl of the vector potential gives us the magnetic field

$$\mathbf{B} = \nabla \times \mathbf{A}$$

## 3. Explain the energy stored by capacitors.

Let us consider charging an initially uncharged parallel plate capacitor by transferring a Charge  $Q$  from one plate to the other, leaving the former plate with charge  $-Q$  and the latter with charge  $+Q$ . Of course, once we have transferred some charge, an electric field is set up between the plates which

opposes any further charge transfer. In order to fully charge the capacitor, we must do work against this field, and this work becomes energy stored in the capacitor. Suppose that the capacitor plates carry a charge  $q$  and that the potential difference between the plates is  $V$ . The work we do in transferring an infinitesimal amount of charge  $dq$  from

the negative to the positive plate is simply

$$dW = V dq$$

In order to evaluate the total work  $W(Q)$  done in transferring the total charge  $Q$  one plate to the other, we can divide this charge into many small increments  $dq$  find the incremental work  $dW$  done in transferring this incremental charge, using the above formula, and then sum all of these works. The only complication is that the potential difference  $V$ , between the plates is a function of the total transferred charge. In fact,  $V(q) = q/C$ . so,  $dW = q dq/C$ . integration yields,

$$W(Q) = \int_0^Q \frac{q dq}{C} = \frac{Q^2}{2C}$$

note again, that the work  $W_m$  done in charging the capacitor is the same as the energy stored in the capacitor. Since  $C = Q/V$ , we can write this stored energy in one of three equivalent forms:

$$W = \frac{Q^2}{2C} = \frac{CV^2}{2} = \frac{QV}{2}$$

#### 4. Write Maxwell's equation in integral form .

For Gauss' law and Gauss' law for magnetism, we've actually already done this. First, we write them in differential form:

$$\begin{aligned} \nabla \cdot E &= \frac{1}{\epsilon_0} \rho \\ \nabla \cdot B &= 0 \end{aligned}$$

We pick any region  $V$  we want and integrate both sides of each equation over that region:

$$\int_V \nabla \cdot E \, dV = \int_V \frac{1}{\epsilon_0} \rho \, dV$$

$$\int_V \nabla \cdot B \, dV = \int_V 0 \, dV$$

On the left-hand sides we can use the divergence theorem, while the right sides can simply be evaluated:

$$\int_{\partial V} E \cdot dS = \frac{1}{\epsilon_0} Q(V)$$

$$\int_{\partial V} B \cdot dS = 0$$

where  $Q(V)$  is the total charge contained within the region  $V$ . Gauss' law tells us that the flux of the electric field out through a closed surface is (basically) equal to the charge contained inside the surface, while Gauss' law for magnetism tells us that there is no such thing as a magnetic charge.

Faraday's law was basically given to us in integral form, but we can get it back from the differential form:

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

We pick any surface  $S$  and integrate the flux of both sides through it:

$$\int_S \nabla \times E \cdot dS = \int_S -\frac{\partial B}{\partial t} \cdot dS$$

On the left we can use Stokes' theorem, while on the right we can pull the derivative outside the integral:

$$\int_{\partial S} E \cdot dr = -\frac{\partial}{\partial t} \Phi_S(B)$$

where  $\Phi_S(B)$  is the flux of the magnetic field  $B$  through the surface  $S$ . Faraday's law tells us that a changing magnetic field induces a current around a circuit.

A similar analysis helps with Ampère's law:

$$\nabla \times B = \mu_0 J + \epsilon_0 \mu_0 \frac{\partial E}{\partial t}$$

We pick a surface and integrate:

$$\int_S \nabla \times B \cdot dS = \int_S \mu_0 J \cdot dS + \int_S \epsilon_0 \mu_0 \frac{\partial E}{\partial t} \cdot dS$$

Then we simplify each side.

$$\int_{\partial S} B \cdot dr = \mu_0 I_S + \epsilon_0 \mu_0 \frac{\partial}{\partial t} \Phi_S(E)$$

where  $\Phi_S(E)$  is the flux of the electric field  $E$  through the surface  $S$ , and  $I_S$  is the total current flowing through the surface  $S$ . Ampère's law tells us that a flowing current induces a magnetic field around the current, and Maxwell's correction tells us that a changing electric field behaves just like a current made of moving charges.

### 5. Explain energy stored in inductor.

Suppose that an inductor of inductance  $L$  is connected to a variable DC voltage supply. The supply is adjusted so as to increase the current  $i$  flowing through the inductor from zero to some final value  $I$ . As the current through the inductor is ramped up, an emf is generated, which acts to oppose the increase in the current. Clearly, work must be done against this emf by the voltage source in order to establish the current in the inductor. The work done by the voltage source during a time interval  $dt$  is

$$dW = P dt = -\mathcal{E} i dt = i L \frac{di}{dt} dt = L i di.$$

Here,  $P = \mathcal{E} i$  is the instantaneous rate at which the voltage source performs work. To find the total work  $W$  done in establishing the final current  $I$  in the inductor, we must integrate the above expression. Thus,

$$W = L \int_0^I i di,$$

giving

$$W = \frac{1}{2} L I^2.$$

This energy is actually stored in the magnetic field generated by the current flowing through the inductor. In a pure inductor, the energy is stored without loss, and is returned to the rest of the circuit when the current through the inductor is ramped down, and its associated magnetic field collapses.

Consider a solenoid

$$W = \frac{1}{2} L I^2 = \frac{\mu_0 N^2 A}{2l} \left( \frac{B l}{\mu_0 N} \right)^2,$$

which reduces to

$$W = \frac{B^2}{2\mu_0} l A.$$

This represents the energy stored in the magnetic field of the solenoid. However, the volume of the field-filled core of the solenoid is  $l A$ , so the magnetic energy density (*i.e.*, the energy per unit volume) inside the solenoid is  $w = W / (l A)$ , or

$$w = \frac{B^2}{2\mu_0}. \quad (252)$$

It turns out that this result is quite general. Thus, we can calculate the energy content of any magnetic field by dividing space into little cubes (in each of which the magnetic field is approximately uniform), applying the above formula to find the energy content of each cube, and summing the energies thus obtained to find the total energy.

When electric and magnetic fields exist together in space, give an expression for the total energy stored in the combined fields per unit volume:

$$w = \frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}.$$

## 6. Derive the expression of Poisson's equation.

Electric potentials can be conveniently related to the charge density by using Laplace equation as follows

The electric field  $E$  is related to the charge density  $\rho$  by the divergence theorem as follows

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

where  $E$  = electric field,  $\rho$  = charge density,  $\epsilon_0$  = permittivity the relationship between electric field and electric potential  $V$  is expressed using gradient as follows

$$E = -\nabla V$$

Electric potential  $V$  is expressed in terms of its source i.e. charge density  $\rho$  as follows

$$\nabla \cdot \nabla V = \nabla^2 V = -\rho / \epsilon_0$$

for Cartesian co-ordinates

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho}{\epsilon_0}$$

This is called as Poisson's equation. Thus charge density  $\rho$  is the divergence of the gradient of a function  $V$ .

## 7. Derive the expression of Laplace equation.

Electric potentials can be conveniently related to the charge density by using Laplace equation as follows

The electric field  $E$  is related to the charge density  $\rho$  by the divergence theorem as follows

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

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Electric potential  $V$  is expressed in terms of its source i.e. charge density  $\rho$  as follows

$$\nabla \cdot \nabla V = \nabla^2 V = -\rho / \epsilon_0$$

for Cartesian co-ordinates

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho}{\epsilon_0}$$

If the region of space is charge- free, i.e.  $\rho = 0$ , then Poisson's equation becomes  $\nabla^2 V = 0$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

The divergence of the gradient of a function, is mathematical operation called as the Laplacian. In general, for a scalar function  $\psi$

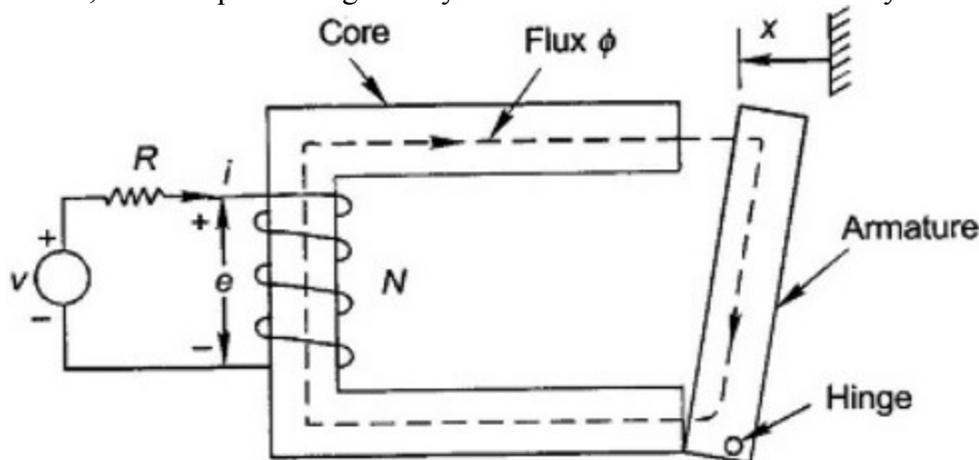
$$\nabla \cdot \nabla \psi = 0$$

Both Poisson's and Laplace's equations are second order partial differential equations. In general they have an infinite number of solutions. The actual solution for a given problem is determined by the boundary conditions. Solutions of Laplace's equation are called harmonic functions. Laplace's equation plays an important role in different branches of physics. The solutions of Laplace's equation express the behavior of different potentials e.g. electric, gravitational, and fluid potentials occurring in electromagnetism, astronomy and fluid dynamics respectively. The Laplacian can be expressed in different coordinate systems so that the symmetry of a charge distribution can be exploited advantageously. This strategy simplifies the solution for the electric potential  $V$ . In addition,  $V$  being a scalar function, its evaluation is simpler than that of  $E$  which is a vector quantity. Once the potential has been calculated, the electric field can be computed by taking the gradient of the potential. For example, if the charge distribution has spherical symmetry, it is convenient to use the Laplacian in spherical polar coordinates.

## 8.Explain energy functional.

Energy in Magnetic System –The chief advantage of electric energy over other forms of energy is the relative ease and high efficiency with which it can be transmitted over long distances. Its main use is in the form of a transmitting link for transporting other forms of energy, e.g. mechanical, sound, light, etc. from one physical location to another. Electric energy is seldom available naturally and is rarely directly utilized. Obviously two kinds of energy conversion devices are needed—to convert one form of energy to the electric form and to convert it back to the original or any other desired form. Our interests in this chapter are the devices for electromechanical energy conversion. These devices can be transducers for low-energy conversion processing and transporting. These devices can be transducers for processing and transporting low-energy signals. A second category of such devices is meant for production of force or torque with limited mechanical motion like electromagnets, relays, actuators, etc. A third category is the continuous energy conversion devices like motors or generators which are used for bulk energy conversion and utilization.

Electromechanical energy conversion takes place via the medium of a magnetic or electric field—the magnetic field being most suited for practical conversion devices. Because of the inertia associated with mechanically moving members, the fields must necessarily be slowly varying, i.e. quasistatic in nature. The conversion process is basically a reversible one though practical devices may be designed and constructed to particularly suit one mode of conversion or the other. Energy can be stored or retrieved from a magnetic system by means of an exciting coil connected to an electric source. Consider, for example the magnetic system of an attracted armature relay



The resistance of the coil is shown by a series lumping outside the coil which then is regarded as an ideal loss-less coil. The coil current causes magnetic flux to be established in the magnetic circuit. It is assumed that all the flux  $\phi$  is confined\* to the iron core and therefore links all the  $N$  turns creating the coil  $I$  flux linkages of

$$\lambda = N \phi$$

the emf is given by  $e = \frac{d\lambda}{dt}$

The electric energy input into the ideal coil due to the flow of current  $I$  in time  $dt$  is

$$dW_e = e i dt$$

Assuming for the time being that the armature is held fixed at position  $x$ , all the input energy is stored in the magnetic field. Thus

$$dW_e = e i dt = dW_f$$

where  $dW_f$  is the change in field energy in time  $dt$ .

$$dW_e = I d\lambda = dW_f$$

The energy absorbed by the magnetic system to establish flux  $\Phi$  (or flux linkages  $\Lambda$ ) from initial zero flux is

$$W_f = \int_0^\Lambda i(\lambda) d\lambda = \int_0^\Phi \mathcal{F}(\phi) d\phi$$

This is the energy of the magnetic field with given mechanical configuration when its state corresponds to flux  $\Phi$  (or flux linkages  $\Lambda$ ).

### 9. Explain electric scalar potential.

Suppose that  $\mathbf{r} = (x, y, z)$  and  $\mathbf{r}' = (x', y', z')$  in Cartesian coordinates. The  $x$  component of  $(\mathbf{r} - \mathbf{r}')/|\mathbf{r} - \mathbf{r}'|^3$  is written

$$\frac{x - x'}{[(x - x')^2 + (y - y')^2 + (z - z')^2]^{3/2}}$$

However, it is easily demonstrated that

$$\frac{x - x'}{[(x - x')^2 + (y - y')^2 + (z - z')^2]^{3/2}} = -\frac{\partial}{\partial x} \left( \frac{1}{[(x - x')^2 + (y - y')^2 + (z - z')^2]^{1/2}} \right)$$

Since there is nothing special about the  $x$ -axis, we can write

$$\frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} = -\nabla \left( \frac{1}{|\mathbf{r} - \mathbf{r}'|} \right),$$

$$\nabla \equiv (\partial/\partial x, \partial/\partial y, \partial/\partial z)$$

where is a differential operator which involves the components of  $\mathbf{r}$  but not those of  $\mathbf{r}'$ .

$$\mathbf{E} = -\nabla\phi,$$

$$\text{Where } \phi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}'.$$

Thus, the electric field generated by a collection of fixed charges can be written as the gradient of a scalar potential, and this potential can be expressed as a simple volume integral involving the charge distribution. The scalar potential generated by a charge  $q$  located at the origin is

$$\phi(r) = \frac{q}{4\pi \epsilon_0 r}.$$

The scalar potential generated by a set of  $N$  discrete charges  $q_i$ , located at  $r_i$ , is

$$\phi(\mathbf{r}) = \sum_{i=1}^N \phi_i(\mathbf{r}),$$

where

$$\phi_i(\mathbf{r}) = \frac{q_i}{4\pi \epsilon_0 |\mathbf{r} - \mathbf{r}_i|}.$$

Thus, the scalar potential is just the sum of the potentials generated by each of the charges taken in isolation. Suppose that a particle of charge  $q$  is taken along some path from point  $P$  to point  $Q$ . The net work done on the particle by electrical forces is

$$W = \int_P^Q \mathbf{f} \cdot d\mathbf{l},$$

where  $\mathbf{f}$  is the electrical force, and  $d\mathbf{l}$  is a line element along the path.

$$W = q \int_P^Q \mathbf{E} \cdot d\mathbf{l} = -q \int_P^Q \nabla \phi \cdot d\mathbf{l} = -q [\phi(Q) - \phi(P)].$$

Thus, the work done on the particle is simply minus its charge times the difference in electric potential between the end point and the beginning point. This quantity is clearly independent of the path taken between  $P$  and  $Q$ . So, an electric field generated by stationary charges is an example of a conservative field. The electric field is the gradient of a scalar potential. The work done on the particle when it is taken around a closed loop is zero, so

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = 0$$

for any closed loop  $C$ . This implies from Stokes' theorem that

$$\nabla \times \mathbf{E} = 0$$

for any electric field generated by stationary charges. since  $\nabla \times \nabla \phi = 0$  for any scalar potential  $\phi$ . The SI unit of electric potential is the volt, which is equivalent to a joule per coulomb. Thus, according to the electrical work done on a particle when it is taken between two points is the product of its charge and the voltage difference between the points. We are familiar with the idea that a particle moving in a gravitational field possesses potential energy as well as kinetic energy. If the particle moves from point  $P$  to a lower point  $Q$  then the gravitational field does work on the particle causing its kinetic energy to increase.

The increase in kinetic energy of the particle is balanced by an equal decrease in its potential energy, so that the overall energy of the particle is a conserved quantity. Therefore, the work done on the particle as it moves from  $P$  to  $Q$  is *minus* the difference in its gravitational potential energy between points  $Q$  and  $P$ . Of course, it only makes sense to talk about gravitational potential energy because the gravitational field is conservative.

Thus, the work done in taking a particle between two points is path independent, and, therefore, well-defined. This means that the difference in potential energy of the particle between the beginning and end points is also well-defined. We have already seen that an electric field generated by stationary charges is a conservative field. It follows that we can define an electrical potential energy of a particle

moving in such a field. By analogy with gravitational fields, the work done in taking a particle from point P to point Q is equal to minus the difference in potential energy of the particle between points Q and P. It follows from the potential energy of the particle at a general point Q, relative to some reference point P (where the potential energy is set to zero), is given by

$$\mathcal{E}(Q) = q\phi(Q).$$

Free particles try to move down gradients of potential energy, in order to attain a minimum potential energy state. Thus, free particles in the Earth's gravitational field tend to fall downwards. Likewise, positive charges moving in an electric field tend to migrate towards regions with the most negative voltage, and *vice versa* for negative charges. The scalar electric potential is undefined to an additive constant. So, the transformation

$$\phi(\mathbf{r}) \rightarrow \phi(\mathbf{r}) + c$$

The potential can be fixed unambiguously by specifying its value at a single point. The usual convention is to say that the potential is zero at infinity, where it can be seen that  $\phi \rightarrow 0$  as  $|\mathbf{r}| \rightarrow \infty$ , provided that the total charge  $\int \rho(\mathbf{r}') d^3\mathbf{r}'$  is finite.

### 10. Deduce the expression for energy stored in spherical capacitance.

Two concentric spherical conducting shells are separated by vacuum. The inner shell has total charge +Q and outer radius  $r_a$ , and outer shell has charge -Q and inner radius  $r_b$ . There are two ways to solve the problem – by using the capacitance, by integrating the electric field density. Using the capacitance, (The capacitance of a spherical capacitor is derived in Capacitance Of Spherical Capacitor.

$$C = 4\pi\epsilon_0 \frac{r_a r_b}{r_b - r_a}$$

$$U = \frac{Q^2}{2C}$$

$$= \frac{Q^2}{8\pi\epsilon_0} \frac{r_b - r_a}{r_a r_b}$$

$$= \frac{Q^2}{8\pi\epsilon_0} \left( \frac{1}{r_a} - \frac{1}{r_b} \right)$$

Now, the integration of electric field energy density method.

Electric field in the volume between the two conducting spheres:

Energy density,

$$\begin{aligned}
 u &= \frac{1}{2} \epsilon_0 E^2 \\
 &= \frac{Q^2}{32\pi^2 \epsilon_0 r^4} \\
 U &= \int u dV \\
 &= \int_{r_a}^{r_b} u 4\pi r^2 dr \\
 &= \frac{Q^2}{8\pi \epsilon_0} \int_{r_a}^{r_b} \frac{1}{r^2} dr \quad E = \frac{Q}{4\pi \epsilon_0 r^2} \\
 &= \frac{Q^2}{8\pi \epsilon_0} \left( \frac{1}{r_a} - \frac{1}{r_b} \right)
 \end{aligned}$$

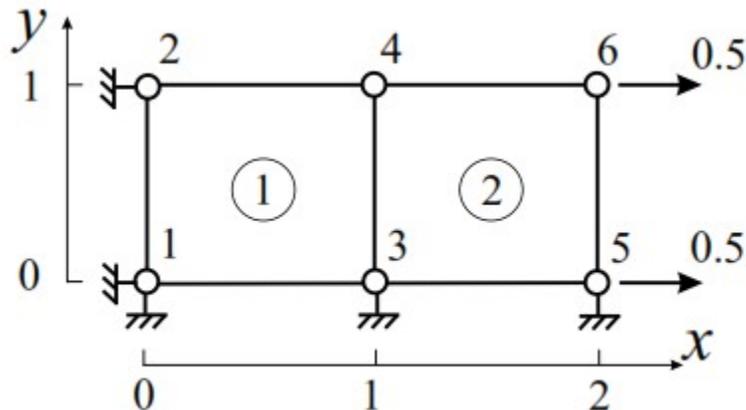
## UNIT-III PHILOSOPHY OF FEM

### 1. Discuss in detail the concept behind the finite element discretization.

In order to apply finite element procedures a discrete model of the problem should be presented in numerical form. A typical description of the problem can contain: Scalar parameters (number of nodes, number of elements etc.); Material properties; Coordinates of nodal points; Connectivity array for finite elements; Arrays of element types and element materials; Arrays for description of displacement boundary conditions; Arrays for description of surface and concentrated loads; Temperature field. Let us write down numerical information for a simple problem depicted in Fig. The finite element model can be described as follows:

#### 1. Scalar parameters

Number of nodes = 6  
 Number of elements = 2  
 Number of constraints = 5



Number of loads = 2

#### 2. Material properties

Elasticity modulus = 2.0e+8 MPa Poisson's ratio = 0.3

#### 3. Node coordinates ( $x_1, y_1, x_2, y_2$ etc.)

1) 0 0 2) 0 1 3) 1 0 4) 1 1 5) 2 0 6) 2 1

4. Element connectivity array (counterclockwise direction)

1) 1 3 4 2 2) 3 5 6 4

5. Constraints (node, direction:  $x = 1$ ;  $y = 2$ )

1 1 2 1 1 2 3 2 5 2

6. Nodal forces (node, direction, value)

5 1 0.5 6 1 0.5. While for simple example like the demonstrated above the finite element model can be coded by hand, it is not practical for real-life models. Various automatic mesh generators are used for creating finite element models for complex shapes.

## 2. Discuss in detail 2D planar.

### 2D Plane Problem:

2D plane problem is a simplification of a 3D problem. It is widely used due to its efficiency.

### Plane Stress and Plane Strain:

**Plane stress:** All the stress components associate with 3-direction are zero.

$$\sigma_{33} = \sigma_{13} = \sigma_{23} = 0 \quad \sigma_{11} \neq 0 \quad \sigma_{22} \neq 0 \quad \sigma_{12} \neq 0$$

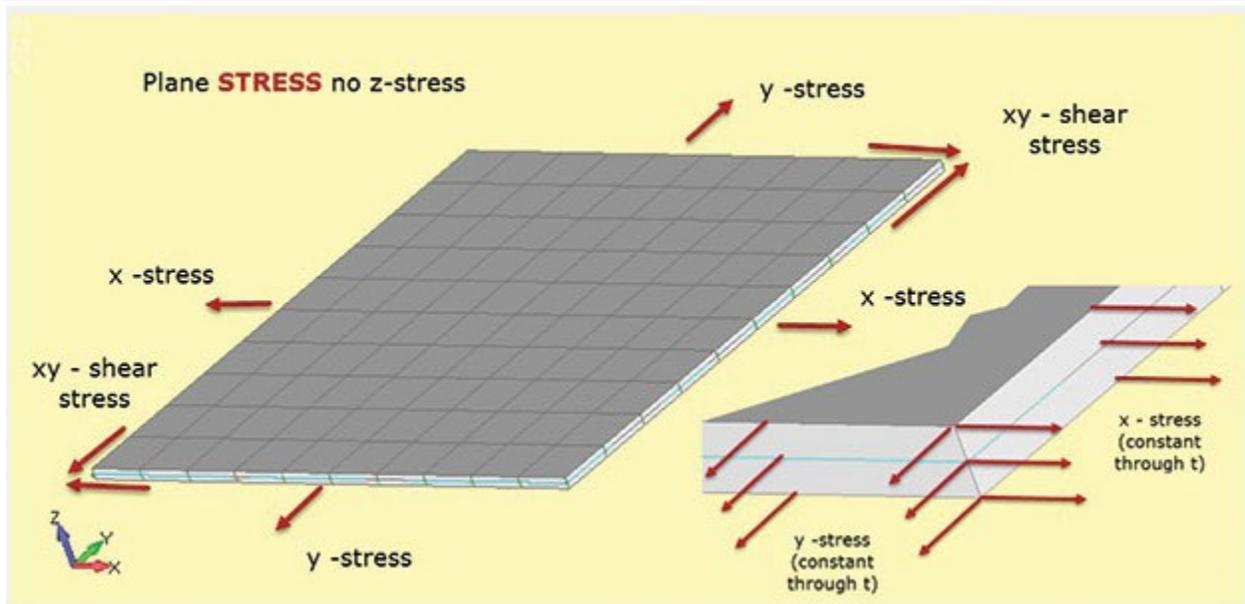
$$F_3 = 0$$

**Plane strain:** All the strain components associate with 3-direction are zero.

$$\varepsilon_{33} = \varepsilon_{13} = \varepsilon_{23} = 0 \quad \varepsilon_{11} \neq 0 \quad \varepsilon_{22} \neq 0 \quad \varepsilon_{12} \neq 0$$
$$u_3 = 0$$

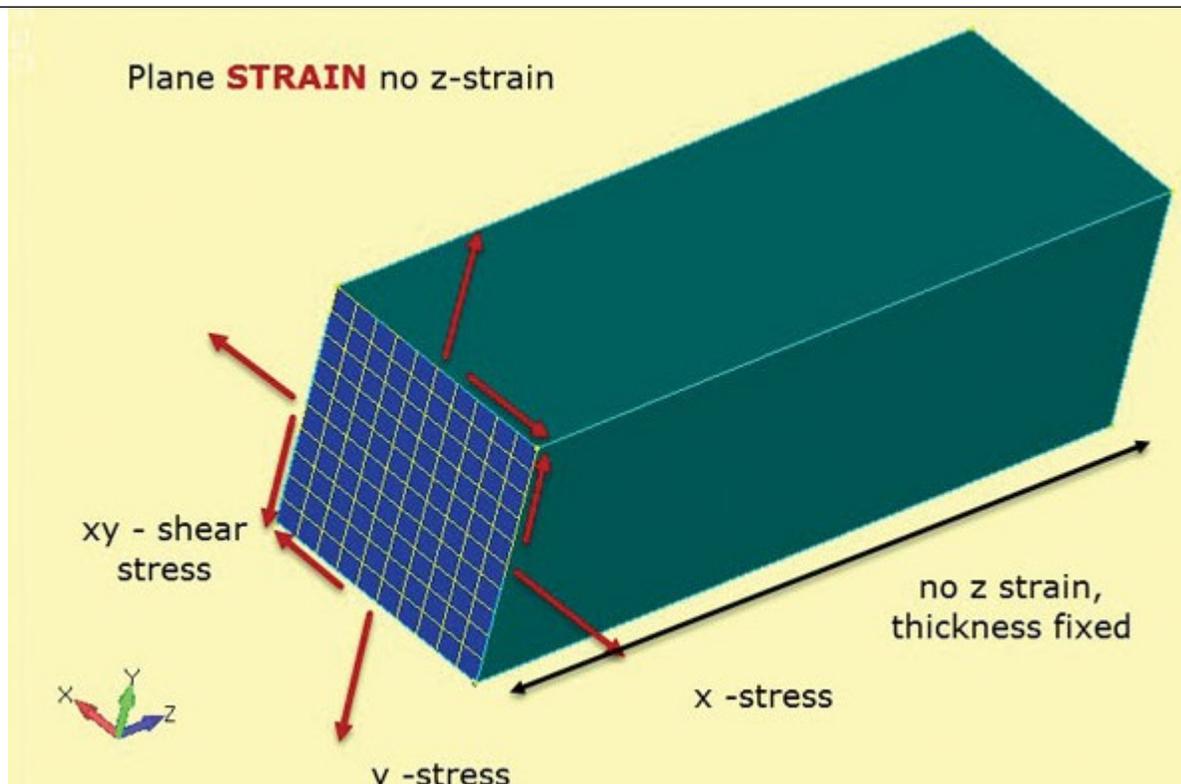
### PLANE STRESS ANALYSIS:

- The structural region is assumed to lie in the 2D xy plane, with the third structural dimension relatively small.
- In the figure, this is the thickness in the z direction. Stresses exist in the 2D plane as sigma x, sigma y (direct stresses) and sigma xy (in-plane shear stress).
- Each of these stresses is constant through the thickness as shown in the inset. In addition there can be no stress in the z direction.
- This stress-strain material relationship is defined in 2D plane stress elements used in this type of analysis.



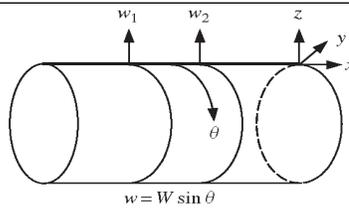
### PLANE STRAIN ANALYSIS:

- 2D planar elements are used, but with subtly different assumptions.
- The in-plane stresses  $x$ ,  $y$  and  $xy$  are developed as before. However this time it is the out-of-plane, or through thickness  $z$  strain which is set to zero.
- So plane strain analysis only allows strains in-plane. This works well to represent thick structures such as shown.
- The presence of this much material tends to stabilize the component and prevent it straining in  $z$ . This also means that constant through thickness  $z$  stresses are developed in the structure.
- This stress-strain material relationship is defined in 2D plane strain elements used in this type of analysis.

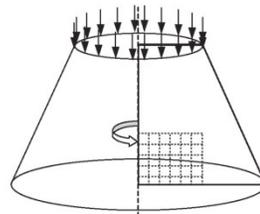


### 3.Explain axial symmetry problems.

- A solid or structure is said to have axial symmetry when the solid can be generated by rotating a planar shape about an axis. Hence, such a solid can be modelled by simply using a special type of 2D or 1D element, called an axisymmetric element.
- In this way, a 3D solid can be modelled simply by using 1D or 2D elements that will greatly reduce the modelling and computational effort. For example, a cylindrical shell structure can be modelled using 1D axisymmetric beam elements, as shown

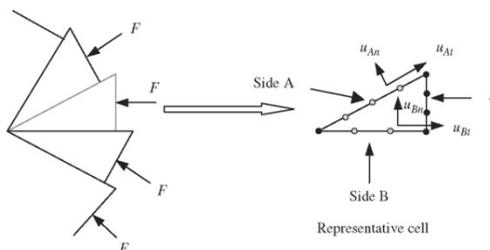


**A cylindrical shell structure modelled using 1D axisymmetric elements.**



**A 3D structure modelled using 2D axisymmetric elements.**

- **The formulation of 1D or 2D axial symmetric elements** is much similar to the 1D or 2D elements developed in earlier topics, except that all the equations need to be expressed in the polar coordinate system instead of the Cartesian coordination system. Generally speaking, the use of axisymmetric elements requires fewer computational resources compared to a full 3D discretization. Axisymmetric elements are readily available in most finite element software packages, and the use of these elements is similar to their counterpart of regular 1D or 2D elements.
- **Similar to the plane symmetry problems**, the loadings applied on an axial symmetric structure do not have to be axial symmetric or axial anti-symmetric. Any axial asymmetric load can be expressed in a Fourier superimposition of both axial symmetric and axial antisymmetric components in the  $\theta$  direction (see Figure 11.23). Therefore, the problem can always be decomposed into two sets of axial symmetric and axial anti-symmetric problems, as long as the structure is axial symmetric (in geometry, material and boundary support).



**Representative cell isolated from a cyclic symmetric structure and the cyclic symmetrical conditions on the cell.**

#### **4.Explain variational method.**

- Rayleigh–Ritz method is a direct method for minimizing a given functional. It is direct in the sense that it yields a solution to the variational problem without solving the associated Euler-Lagrange Equation.
- It may be noted that, for most of the physical problems, the functional we get from the variational principle is not simple and thus the solution using the EL equation will be difficult to obtain.
- The Rayleigh–Ritz method is an approximate method where the given functional is directly minimized without recourse to the associated EL equation.
- To illustrate the method let us consider the following functional

$$J[\varphi] = \int_{D_s} ZS F(x, y, \varphi, \varphi_x, \varphi_y) Ds$$

Our objective is to minimize this integral. In the Rayleigh-Ritz method, we select a linearly independent set of functions called basis functions  $u_n$  and construct an approximate solution to equation satisfying some prescribed boundary conditions. The solution is in the form of a finite series

$$\tilde{\varphi} = u_0 + \sum_{n=1}^N a_n u_n$$

Where  $u_0$  meets the non homogeneous boundary conditions if any, and  $u_n$  satisfies homogeneous boundary conditions. The unknown coefficients are to be determined and  $\tilde{\varphi}$  is an approximate solution to the exact solution  $\varphi$ . Substitution of the approximate solution in the function with  $N$  coefficients  $a_1, a_2, \dots, a_N$ . That is,

$$J(\tilde{\varphi}) = J(a_1, a_2, \dots, a_N)$$

The minimum of this function is obtained when its partial derivatives with respect to each coefficient is zero. That is,

$$\frac{\partial J}{\partial a_1} = 0, \quad \frac{\partial J}{\partial a_2} = 0, \quad \dots \quad \frac{\partial J}{\partial a_N} = 0$$

Thus we obtain a system of  $N$  linear algebraic equations which can be solved to obtain  $a_n$ . These  $a_n$  are then substituted into the approximate solution. Now, if  $\tilde{\varphi} \rightarrow \varphi$  as  $N \rightarrow \infty$  in some sense, then the procedure is said to converge to the exact solution. The basis functions are selected to satisfy the prescribed boundary conditions of the problem.  $u_0$  is chosen to satisfy the inhomogeneous boundary conditions, while  $u_n$  ( $n=1, 2, \dots, N$ ) are selected to satisfy the homogeneous boundary conditions.

## 5. Explain finite difference method.

- The finite difference method is the easiest method to understand and apply. To solve a differential equation using finite difference method, first a mesh or grid will be laid over the domain of interest. This process is called the discretization.
- A typical grid point in the mesh may be designated as  $i$ . The next step is to replace all derivatives present in the differential equation by suitable algebraic difference quotients.
- For example, the derivative  $\frac{d\phi}{dx}$ , may be approximated as a first-order accurate forward difference quotient

$$\left. \frac{d\phi}{dx} \right|_i \approx \frac{\phi_{i+1} - \phi_i}{\Delta x}$$

or as a second-order accurate central difference quotient

$$\left. \frac{d\phi}{dx} \right|_i \approx \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x}$$

Where  $\Delta x$  is the grid size and  $\phi_i$  is the value of  $\phi$  at  $i$ th grid point and is an unknown.

- This process yields an algebraic equation for the typical grid point  $i$ .
- The application of the algebraic equation to all interior grid points will generate a system of algebraic equations in which the grid point values of  $\phi$  are unknowns.
- After the introduction of proper boundary conditions, the number of unknowns in the equation will be equal to the number of interior nodes in the mesh.
- The system (of equations) is typically solved using iterative methods such as Jacobi method, Gauss-Seidel method, or any of the advanced techniques.
- We note that the finite difference method gives point-wise approximation to the differential equation and hence it gives the values of dependent variables at discrete points. Using finite difference approach we can solve fairly difficult problems.
- It works well when the boundaries of the domain are parallel to the coordinate axes. But, we find that the method becomes harder to use when irregular boundaries are encountered.
- It is also difficult to write general purpose computer codes for FDM.

## 6.Explain finite element method.

- The finite element method is a very versatile numerical technique and is a general purpose tool to solve any type of physical problems.
- It can be used to solve both field problems (governed by differential equations) and non-field problems.
- There are several advantages of FEM over FDM. Among them, the most important advantage is that FEM is well suited for problem with complex geometries, because no special difficulties are encountered when the physical domain has a complex geometry.
- The other important advantage is that it is easier to write general purpose computer codes for FEM formulations.

Three different approaches are being used when formulating an FEM problem. They are:

1. Direct Approach
2. Variational Approach
3. Weighted Residual Method

### **Direct Approach:**

The direct approach is related to the “direct stiffness method” of structural analysis and it is the easiest to understand when meeting FEM for the first time. The main advantage of this approach is that you can get a feel of basic techniques and the essential concept involved in the FEM formulation without using much of mathematics. However, by direct approach we can solve only simple problems.

### **Variational Approach:**

In variational approach the physical problem has to be restated using some variational principle such as principle of minimum potential energy. It is widely used for deriving finite element equations whenever classical variational statement is available for the given problem. A basic knowledge of calculus of variations is required to use variational approach. The major disadvantage of the variational approach is that there exist many physical problems for which classical variational statement may not be available. This is the case with most of the nonlinear problems. In such cases variational approach is not useful. The Rayleigh-Ritz method is an approximate method based on the variational formulation.

### **Weighted Residual Method:**

Weighted residual method (WRM) is a class of method used to obtain the approximate solution to the differential equations of the form

$$\mathcal{L}(\phi) + f = 0 \quad \text{in } D$$

In WRM, we directly work on differential equation of the problem without relying on any variational principle. It is equally suited for linear and nonlinear differential equations. Weighted residual method involves two major steps. In the first step, we assume an approximate solution based on the general behavior of the dependent variable. The approximate solution is so selected that it satisfies the boundary conditions for  $\phi$ . The assumed solution is then substituted in the differential equation.

## 7. Discuss shape function.

- The shape function is the function which interpolates the solution between the discrete values obtained at the mesh nodes.
- Therefore, appropriate functions have to be used and, as already mentioned, low order polynomials are typically chosen as shape functions. In this work linear shape functions are used.
- For three-dimensional finite element simulations it is convenient to discretize the simulation domain using tetrahedrons, The linear shape function of the node  $i$  has the form

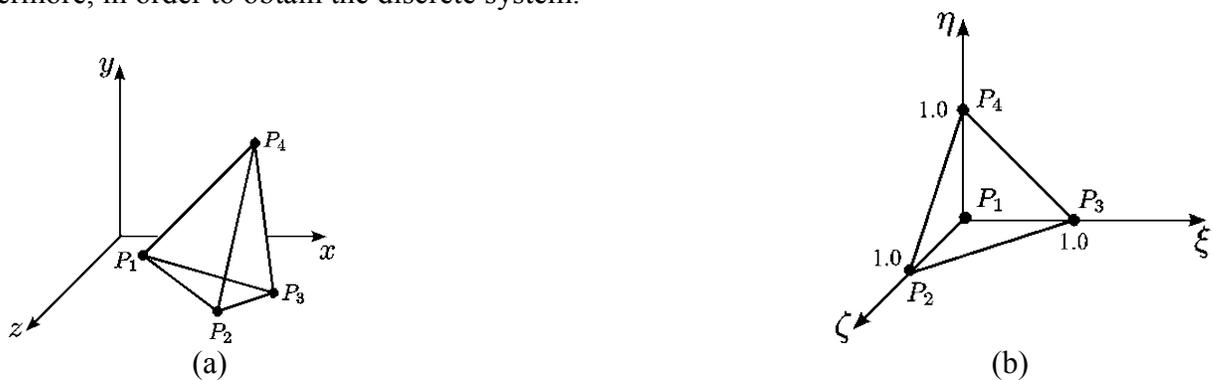
$$N_i(x, y, z) = a_i + b_i x + c_i y + d_i z,$$

where  $i=1,2,\dots$ . The coefficients,  $a_i, b_i, c_i$  and  $d_i$  for each nodal basis function of the tetrahedral element

$$N_j(\vec{r}_i) = \delta_{ij}, \quad i, j = 1, \dots, 4.$$

can be calculated considering the condition.

As a result, a system of 4 equations for the 4 unknown coefficients is obtained. This procedure has to be repeated for all tetrahedrons of the mesh, so that the basis functions of all grid nodes are determined. Furthermore, in order to obtain the discrete system.



**Figure:** Tetrahedral finite element. (a) Original coordinate system. (b) Transformed coordinate system.

The calculations can be significantly simplified by carrying out a coordinate transformation. A tetrahedron in a transformed coordinate system is shown in Figure (b). Each point  $(x,y,z)$  of the tetrahedron in the original coordinate system can be mapped to a corresponding point  $(\xi, \eta, \zeta)$  in the transformed coordinate system.

$$x = x_1 + (x_2 - x_1)\xi + (x_3 - x_1)\eta + (x_4 - x_1)\zeta,$$

$$y = y_1 + (y_2 - y_1)\xi + (y_3 - y_1)\eta + (y_4 - y_1)\zeta,$$

$$z = z_1 + (z_2 - z_1)\xi + (z_3 - z_1)\eta + (z_4 - z_1)\zeta,$$

which in matrix form leads to the Jacobian matrix

$$\mathbf{J} = \begin{bmatrix} x_2 - x_1 & x_3 - x_1 & x_4 - x_1 \\ y_2 - y_1 & y_3 - y_1 & y_4 - y_1 \\ z_2 - z_1 & z_3 - z_1 & z_4 - z_1 \end{bmatrix}.$$

In this way, the nodal basis functions for the tetrahedron in the transformed coordinate system are given by

$$N_1^t(\xi, \eta, \zeta) = 1 - \xi - \eta - \zeta,$$

$$N_2^t(\xi, \eta, \zeta) = \xi,$$

$$N_3^t(\xi, \eta, \zeta) = \eta,$$

$$N_4^t(\xi, \eta, \zeta) = \zeta.$$

These shape functions are rather simple, so that the derivatives and integrals required for the finite element formulation can be readily evaluated in the transformed coordinate system. Given a function  $f(x,y,z)$ , the gradient in the transformed coordinates is of the form

$$\nabla^t f = \left[ \frac{\partial f}{\partial \xi} \quad \frac{\partial f}{\partial \eta} \quad \frac{\partial f}{\partial \zeta} \right]^T,$$

where the derivatives are calculated via the chain rule by

$$\frac{\partial f}{\partial \xi} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial \xi} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial \xi} + \frac{\partial f}{\partial z} \frac{\partial z}{\partial \xi},$$

$$\frac{\partial f}{\partial \eta} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial \eta} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial \eta} + \frac{\partial f}{\partial z} \frac{\partial z}{\partial \eta},$$

$$\frac{\partial f}{\partial \zeta} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial \zeta} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial \zeta} + \frac{\partial f}{\partial z} \frac{\partial z}{\partial \zeta}.$$

These equations can be expressed in matrix notation as

$$\begin{bmatrix} \frac{\partial f}{\partial \xi} \\ \frac{\partial f}{\partial \eta} \\ \frac{\partial f}{\partial \zeta} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix},$$

or

$$\nabla^t f = \mathbf{J}^T \nabla f,$$

where  $\mathbf{J}^T$  is the transpose of the Jacobian matrix. Thus, the gradient in the original coordinate system can be calculated using the transformed coordinate gradient by

$$\nabla f = (\mathbf{J}^T)^{-1} \nabla^t f = \mathbf{\Lambda} \nabla^t f,$$

$$\mathbf{\Lambda} = (\mathbf{J}^T)^{-1}$$

Performing such a coordinate transformation significantly simplifies the practical implementation of the FEM. The nodal shape functions in the transformed coordinates are fixed and known in advance, thus, it is not necessary to solve the system of equations for each element of the mesh. Only the Jacobian matrix has to be determined, and the required calculations for the finite element formulation can be easily evaluated.

## 8. Explain stiffness matrix.

- The stiffness matrix is the n-element square matrix A defined by. By defining the vector F with components  $F_i =$ , the coefficients  $u_i$  are determined by the linear system  $AU = F$ .
- The stiffness matrix is symmetric, i.e.  $A_{ij} = A_{ji}$ , so all its eigenvalues are real. In the finite element method for the numerical solution of elliptic partial differential equations, the stiffness matrix represents the system of linear equations that must be solved in order to ascertain an approximate solution to the differential equation.

Comparing the 2-D element stiffness matrix with the 1-D truss stiffness matrix, a pattern emerges:

2D element  $\longrightarrow$  1D element

**2 x 2 color block  $\Leftarrow$ equivalent $\Rightarrow$  1 x 1 color block**

$$[K] = \begin{bmatrix} k_e \begin{bmatrix} CC & CS \\ CS & SS \end{bmatrix} & -k_e \begin{bmatrix} CC & CS \\ CS & SS \end{bmatrix} \\ -k_e \begin{bmatrix} CC & CS \\ CS & SS \end{bmatrix} & k_e \begin{bmatrix} CC & CS \\ CS & SS \end{bmatrix} \end{bmatrix} \quad [K] = \begin{bmatrix} k_e & -k_e \\ -k_e & k_e \end{bmatrix}$$

Where:  $k_e = EA/L$ ,  $CC = \cos^2(\theta)$ ,  $CS = \cos(\theta) * \sin(\theta)$ ,  $SS = \sin^2(\theta)$

Therefore, the 2D element stiffness matrix is comprised of one 2 x 2 block matrix [kb] arranged so that:

$$[K] = \begin{bmatrix} [k_b] & -[k_b] \\ -[k_b] & [k_b] \end{bmatrix}$$

where  $CC = \cos^2(\theta)$ ,  $CS = \cos(\theta) * \sin(\theta)$ ,  $SS = \sin^2(\theta)$   
and the 2 x 2 block matrix [kb] is defined by:

$$[k_b] = k_e \begin{bmatrix} \cos^2(\theta) & \cos(\theta) * \sin(\theta) \\ \cos(\theta) * \sin(\theta) & \sin^2(\theta) \end{bmatrix}$$

## 9. Compare FDM and FEM.

### FINITE DIFFERENCE METHOD:

- FDM is created from basic definition of differentiation that is

$$df/dx=(f(x+h)-f(x))/h$$

here "h" tends to zero.

- In numerical analysis, its not possible to divide a number by "0" so "zero" means a small number.
- So FDM is similar to differential calculus but it has killed the heart that is limit tends to "zero".  
So in most of the cases accuracy of FDM increases with refining grid.
- Easy method but not reliable for conservative differential equations and solutions having shocks.
- Tough to implement in complex geometry where it needs complex mapping and mapping makes governing equation even tougher.
- Extending to higher order accuracy is very simple

### FINITE ELEMENT METHOD:

- It is a numerical tool that is borrowed from calculus of variation.
- There are lot of types of FEM like point collocation method, sub-domain method etc. Here they assume some trial function and multiply that trial function with weighting function .
- In Galerkins method the trial function itself weighting function. Different methods follow different ways in weighting. Then this weighting function is multiplied with trial function then integrated over the control volume ( weak form) and equated to zero (This procedure will differ for different types of FEM but theme is same). Then we get one set of algebraic equations.
- Solving that will give solution. Here we are working only in error and differential equation some times conservative law may be violated.
- This method is more accurate than FVM and FDM. Ideal for linear PDEs, expensive and complex for non-linear PDEs.
- Here higher order accuracy is achieved by using higher order basis (i.e) shape functions. Extending to higher order accuracy is relatively complex than FVM and FDM.
- Higher order accurate calculations are expensive in computation and Mathematical formulation especially for non-linear PDEs.
- Mostly suitable for Heat transfer, Structural mechanics, vibrational analysis etc.
-

## 10. Explain direct integration method.

The simplest differential equations to solve are those in the form of  $dy/dt=f(t)$ . These differential equations can be solved by directly integrating both sides to get that for any antiderivative  $F$  and for  $C$  as a constant we have that:

$$\int \frac{dy}{dt} dt = \int f(t) dt$$
$$y = F(t) + C$$

For example, suppose that we wanted to solve the differential equation  $dy/dt=t^2+\cos 2t$ . Then we have that for  $C$  as a constant:

$$\int \frac{dy}{dt} dt = \int t^2 + \cos 2t dt$$
$$y = \frac{t^3}{3} + \sin 2t + C$$

Another type of differential equation that can be solved by direct substitution are differential equation for which  $a$  and  $b$  are constants, in the following form:

$$\frac{dy}{dt} = ay + b$$

For  $a \neq 0$  and  $y \neq -b/a$  we can rewrite the differential equation above as:

$$\frac{dy}{dt} = a \left( y + \frac{b}{a} \right)$$
$$\frac{\left( \frac{dy}{dt} \right)}{y + \frac{b}{a}} = a$$

Notice that if  $p(t) = \ln \left| y + \frac{b}{a} \right|$  then  $\frac{dp}{dt} = \frac{d}{dt} \ln \left| y + \frac{b}{a} \right| = \frac{\left( \frac{dy}{dt} \right)}{y + \frac{b}{a}}$  and so we have:

$$\frac{d}{dt} \ln \left| y + \frac{b}{a} \right| = a$$
$$\int \frac{d}{dt} \ln \left| y + \frac{b}{a} \right| dt = \int a dt$$
$$\ln \left| y + \frac{b}{a} \right| = at + C$$
$$\left| y + \frac{b}{a} \right| = e^{at+C}$$
$$y + \frac{b}{a} = \pm e^{at+C}$$
$$y = \pm e^{at+C} - \frac{b}{a}$$
$$y = \pm e^{at} e^C - \frac{b}{a}$$

Generally, since  $C$  is an arbitrary constant, we have that  $\pm e^C$  is also an arbitrary constant which we will denote as  $D$ , and hence, the solutions to  $dy/dt=ay+b$  are given by  $y=De^{at}-ba$ . Notice that we must proceed with some caution, since we must check if  $D=0$  yields a solution. In this case, since if  $D=0$

then  $y=-ba$  which was omitted earlier. In most cases,  $y=-ba$  will be a solution to the differential equation.

## UNIT-IV CAD PACKAGES

### 1. Discuss basic elements of CAD.

A set of methods and tools to assist product designers in creating a geometrical representation of the artifacts they are

- Designing.
- Dimensioning.
- Tolerancing.
- Configuration Management (Changes).
- Archiving.
- Exchanging part and assembly information between teams, organizations
- Feeding subsequent design steps.
- Analysis.
- Manufacturing (CAM).

Elements of computer-aided design and manufacturing, CAD. This compact, up-to-date survey of CAD software and hardware presents the principles of interactive graphics and discusses the essential elements of computer-aided design and manufacturing.

<b>Input Devices</b>	<b>Main System</b>	<b>Output Devices</b>
Keyboard Mouse Templates Space Ball CAD keyboard	Computer CAD Software Database	Hard Disk Network Printer Plotter

## **2. Discuss in detail the mesh generation in Computed aided design.**

### **Mesh structure:**

- Domain divided into a structured assembly of quadrilateral cells.
- Each interior nodal points is surrounded by exactly the same number of mesh cells.
- Directions within the mesh can be immediately identify by associating a curvilinear co-ordinates system.
- It is possible to immediately identify the nearest neighbours of any node  $j$  on the mesh.
- Computational domain divided into an unstructured assembly of computational cells. The number of cells surrounding a typical interior node is not necessarily constant.
- The nodes and the elements has to be numbered.
- To get the necessary information on the neighbours the numerotation of the nodes wich belong to each element has to be stored.
- The concept of directionality does not exist anymore.

### **Mesh Properties:**

#### 1. Compatible with solver .

- Finite-difference solver requires mesh to follow lines of constant coordinate
- Most finite-element and finite-volume codes are written only for grid elements of certain shapes (e.g., tetrahedron, hexahedral, etc.)

#### 2. Nodes of adjacent mesh elements are the same.

#### 3. Element angles close to 90 degrees(Orthogonality)

- Meshes with angles that are too small or too large lead to inaccurate solutions, ill-conditioned matrices, and slow (or no) convergence of iterative solvers

#### 4. Provides adequate resolution of computed fields

- Meshes must be finer in fluid/thermal boundary layers, near cracks in solids, near joints, within vortex cores, etc.

#### 5. Uses minimum number of elements

- Triangles use twice as many elements as quadrilaterals
- Tetrahedral use six times as many elements as hexahedral

### **Parameters of mesh quality**

- Grid density.
- Adjacent cell length/volume ratios.( $<20\%$ )
- Skewness.( $<0.85$ )
- Mesh refinement through adaption.

- Orthogonality

### **3.Explain Preprocessing in CAD.**

**Preprocessing** also called Meshing is the first step in solving a problem in [Finite Element Analysis](#) Here the entire domain is discretized (divided) into meaningful divisions often called "Elements". These elements form the building block on which the Boundary conditions and external effects are specified.

Many software exist to create a useable Mesh, Some of the popular ones are

1. **Hypermesh** from [Altair](#)
2. **Medina** from [GRM Consulting](#)
3. **ICEM** from [ANSYS](#)

Meshing a domain would consist of the following tasks

1. Defining a domain to Mesh
  - 1.1 Importing a CAD Geometry (Computer Aided Geometry) OR
  - 1.2 Creating your own Geometry OR
  - 1.3 Creating node based elements
2. Selecting the analysis type (Ex: 3-Dimensional, 2-Dimensional, 1-Dimensional)
3. Creating the mesh
4. Choosing the element type
5. Validating the mesh. Check for Errors, Connectivity, Quality etc.

#### **Node**

Nodes forms the building block of any mesh. An element is specified through it's connectivity to a set of nodes. Nodes can be simply thought of as a coordinate which has a label to it. So that when you refer to this label, you mean the coordinate

#### **Steps in Meshing**

1. The first step in meshing would be to select a suitable element to mesh your domain.
2. Let's continue by saying that you have selected a 2Dimensional mesh of 1st order (corner nodes only) mixed mesh containing rectangles an triangles.
3. You could then define an element as the element type that you want to fill the nodes

#### **Element Quality**

- Element quality is very important for a solver to provide you with a reasonable result.
- Element quality is most often defined as a deviation from a perfect element.
- It's important for the user to understand what is Iso-Parametric element
- Order of element. An element can be of either 1st order

- Elements can be classified into many kinds, but the most important classification would be; Structural and Continuum.
- 

#### **4.Explain Postprocessing in CAD.**

- Post processing of finite element data generally requests additional software to organize the output such that it is easily understandable whether the construction is acceptable or not.
- It can include checks on the codes and standards to which the construction must comply e.g. the check of panel stiffened structures. Writing this software is part of the knowledge-based engineering principle.A
- Post Processor is a unique "driver" specific to a CNC machine, robot or mechanism; some machines start at different locations or require extra movement between each operation, the
- Post-Processor works with the CAM software or off-line programming software to make sure the G-Code output or program is correct for a specific machine build.CAM software uses geometry from a CAD model and converts it to G-code.
- The CAM software analyzes the CAD model, determines what tooling and toolpaths will be used to mill the desired features. Doing so requires a CAM post processor that generates the exact g-code dialect used by the machine that is being targeted.
- An instance of such a translation is often referred to as a "post".
- There will be a different “post” for each g-code dialect the CAM software supports. Post Processors usually do not convert g-code from one dialect to the next, rather the “post” uses an intermediate format that captures the G-code commands in a dialect-independent form. Most CAM software accomplishes this with an intermediate format called "CL Data."
- The Post-Processor will alter the program output to suit a specific machine; a "Post" can be used for complex things like producing a proprietary machine language other than G-Code or M-Code, or a Post-Processor maybe used to start a machine from a specific position.
- Another example of use for a Post-Processor would be an ATC (Automatic-Tool-Change) for a CNC, the Post-Processor is required so the correct Tool is collected from the correct location.Some devices connect to the computer using "Serial Communication" and some CNC devices connect using "Parallel Communication", the Post-Processor does not influence the "communication", the Machine Software does.

## 5. Explain modeling technique in CAD.

- The shapes of the various parts of large electrical machine are too complicated for exact analysis of the heat flow in different parts of the machine.
- This situation has led to the use of the water models because there are serious difficulties in air models when measuring windage losses and the relative gas velocity over rotor cooling surfaces.
- Watermodel are preferred to air model in measuring
  - (i) static pressures and (ii) pressure differences, (iii) hydraulic resistance, (iv) flow rate in duct or in the modelling of hollow conductor, (v) flow velocity vector fields over heat transfer surface, and (vi) windage losses.
- Watermodel represents all parts of the machine which could have an essential influence upon the flow of the cooling gas.
- For the mixed flow case, the strength of rotation is identified by the ratio of tangential velocity on the surface of the rotating shaft to the mean axial velocity.
- The ratio, ( $\check{A} = V_o/V_m$ ) is called the rotation ratio. It was found that at a fixed Reynolds number, the Nusselt number increases with an increase in the rotation ratio,  $\check{A}$ .
- Moreover, the effect of  $\check{A}$  on Nu is particularly strong at low Reynolds numbers and it becomes negligible above  $Re = 50,000$  for the highest rotation.
- A physical parameter that correlates the mixed-mode friction coefficient and Nusselt number data by using the normal pure axial flow relation can be defined as the rotation parameter.
- Moreover, it was common to assume that the rotation of inner cylinder does not significantly affect the Nusselt number until the rotation ratio reaches a value about 0.8.
- The speed of rotation covers the range of the Taylor number up to about  $10^6$ , and the range of the Reynolds number based on the axial velocity components and the gap distance can be assumed to be valid up to 7000.
- It was found that rotation does not affect the Nusselt number at low Taylor Numbers and the heat transfer is determined by the axial Reynolds number.
- For a smooth rotor the Taylor number for the onset of vortex flow increase firstly with increasing axial flow, and, after reaching a maximum, the Taylor number appears to decrease slightly with further increases in axial flow.

## **6. Discuss in detail the material properties for designing any electrical apparatus .**

### **Electrical Conducting Materials:**

Materials serving as electrical conductors can be divided into two main groups, namely,

1. High Conductivity Materials: These materials are used for making all types of winding required in electrical machines, apparatus and devices, as well as for transmission and distribution of electric energy. These materials should have the least possible resistivity.

2. High Resistivity Materials: These materials are used for making resistances and heating devices.

### **I. High Conductivity Materials:**

The fundamental requirements to be met by high conductivity materials are

- a. Highest possible conductivity and hence least resistivity
- b. Least possible temperature coefficient of resistance
- c. Adequate mechanical strength, in particular, high tensile strength and elongation characterizing to a certain degree of the flexibility, i.e. absence of brittleness
- d. Rollability and drawability which is important in the manufacture of wires of small and intricate sections
- e. Good weldability and solderability which ensure high reliability and low electrical resistance of the joints
- f. Adequate resistance to corrosion

The following section gives a brief analysis on the values of resistivity, specific weight, density, resistance temperature coefficient, co-efficient of thermal expansion, thermal, conductivity specific heat and tensile strength of conducting materials used in electrical machines.

#### **a. Copper:**

Copper is the most widely used electrical conductor combining, high electrical conductivity with excellent mechanical properties and relative immunity from oxidation and corrosion. It is highly malleable and ductile metal. It can be cast, forged, rolled, drawn, and machined. Mechanical working hardens it but annealing restores it to soft state.

#### **b. Aluminium:**

The application of aluminium is increasing due to the high demand for conductor materials which cannot be met by copper product alone. Therefore, aluminium which is the conductor material next to copper is used. Also aluminium is available in abundance on earth's surface. Pure aluminium

is softer than copper and therefore, can be rolled into thin sheets (foils). Aluminium cannot be drawn into very fine wires on account of its low mechanical strength.

In replacing copper conductors with aluminium ones in electrical machines due account should be taken of their differences in resistivity, density and mechanical strength.

**c. Iron and Steel:**

Steel alloyed with chromium and aluminium is used for making starter rheostats where lightness combined with robustness and good heat dissipation are important considerations. Cast iron is used in the manufacture of resistance grids to be used in the starters of large motors.

**Magnetic Material:**

All magnetic materials possess magnetic properties to a greater or a lesser degree. The magnetic properties of materials are characterized by their relative permeability. In accordance with the value of relative permeability, materials may be divided into three broad classes.

**a. Ferromagnetic materials:** The relative permeabilities of these materials are much greater than unity and these permeability values are dependent upon the magnetizing force.

**b. Paramagnetic materials:** These materials have their relative permeabilities only slightly greater than unity. The value of susceptibility is thus positive for these materials.

**c. Diamagnetic materials:** These materials have their relative permeabilities slightly less than unity. In both Paramagnetic and Diamagnetic materials the value of permeability is independent of the magnetizing force.

## **7.Explain about insulating materials.**

Insulating materials or Insulants are extremely diverse in origin and properties. They are essentially nonmetallic, organic or inorganic, uniform or heterogeneous in composition, natural or synthetic. Many of them are of natural origin as, for example, paper, cloth, paraffin wax and natural resins. Wide use is made of many inorganic insulating materials such as glass, ceramics and mica. Many of the insulating materials are man-made products manufactured in the form of resins, insulating films etc. In recent years wide use is made of new materials whose composition and properties place them in an intermediate position between inorganic and organic substances. These are the synthetic organo-silicon compounds, generally termed as silicones.

### **Electrical Properties of Insulating Materials:**

There are many properties which determine the suitability of a material for use as an insulating material.

1. Resistivity or specific resistance
2. electric strength or breakdown voltage,
3. permittivity and
4. dielectric hysteresis.

An ideal insulating material should have

- (1) high dielectric strength, sustained at elevated temperatures
- (2) high resistivity or specific resistance
- (3) low dielectric hysteresis
- (4) good thermal conductivity
- (5) high degree of thermal stability i.e. it should not deteriorate at high temperatures.

### **Classification of Insulating Materials:**

#### **Class Y:**

This insulation Consists of materials, or combinations of materials, such as cotton, silk and paper without impregnation. Other materials or combinations of materials can be included in this class, if by experience or accepted tests they can be shown to be capable of operating at class Y temperatures.

Examples: Cotton, silk, paper, cellulose, wood etc., neither impregnated nor immersed in oil. Materials of class Y are unsuitable for electrical machines and apparatus as they deteriorate rapidly and are extremely hygroscopic.

#### **Class A:**

This insulation consists of materials or combinations of materials such as cotton, silk and paper when suitably impregnated or coated when immersed in a dielectric liquid such as oil. Other materials or combinations of materials may be included in this class, if by experience or accepted tests they can be shown to be capable of operation at class A temperatures.

Examples: Materials of class Y impregnated with natural resins cellulose esters, insulating oils, etc. Also included in this class are laminated wood, varnished papers.

#### **Class E:**

This insulation consists of materials or combinations of materials which by experience or accepted tests can be shown to be capable of operating at class E temperature (materials possessing a degree of thermal stability allowing them to be operated at a temperature 15 ° C higher than class A materials).

Examples: Synthetic resin enamels, cotton and paper laminated with formaldehyde bonding, etc.

#### **Class B:**

This insulation consists of materials or combinations of materials such as mica, glass, fibre, asbestos, etc., with suitable bonding substances. Other materials or combinations of materials, not necessarily inorganic, may be included in this class, if by experience or accepted tests they can be shown to be capable of operation at class B temperatures. Examples: Mica, glass fibre, asbestos with suitable bonding substances, built up mica, glass fibre, and asbestos laminates

#### **Class F:**

This insulation consists of materials or combinations of materials, such as mica, glass, fibre, asbestos, etc., with suitable bonding substances as well as other materials or combinations of materials, not necessarily inorganic, which by experience or accepted tests can be shown to be capable of operation at class F temperatures (materials possessing a degree of thermal stability allowing them to be operated at a temperature 95° C higher than class B materials). Examples: Materials of class B with bonding materials of higher thermal stability.

#### **Class H:**

This insulation consists of materials, such as silicon elastomer and combination of materials such as mica glass fibre, asbestos, etc., with suitable bonding substances, such as appropriate silicon resins. Other materials or combinations of materials may be included in this class, if by experience or accepted tests they can be shown to be capable of operation at class H temperature. Examples: Glass fibre and asbestos materials, and built up mica, with silicon resins.

#### **Class C:**

This insulation consists of materials or combinations of materials such a mica, porcelain, glass and quartz with or without an inorganic binder. Other materials or combinations of materials may be included in this class, if by experience or accepted tests they can be shown to he capable of operation at temperatures above the class H limit. Specific materials or combinations of materials in

this class will have a temperature limit which is dependent upon their physical, chemical and electrical properties.

### **8.Explain about Periodic boundary conditions.**

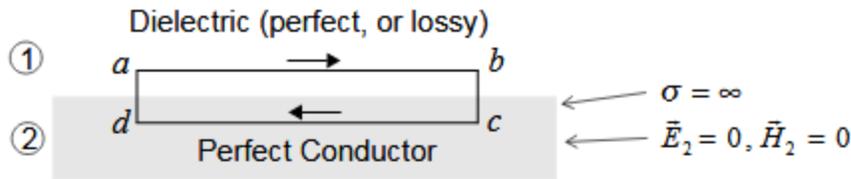
- Periodic boundary conditions (PBCs) are a set of boundary conditions which are often chosen for approximating a large (infinite) system by using a small part called a *unit cell*. PBCs are often used in computer simulations and mathematical models. T
- The topology of two-dimensional PBC is equal to that of a *world map* of some video games; the geometry of the unit cell satisfies perfect two-dimensional tiling, and when an object passes through one side of the unit cell, it re-appears on the opposite side with the same velocity.
- In topological terms, the space made by two-dimensional PBCs can be thought of as being mapped onto a torus (compactification).
- The large systems approximated by PBCs consist of an infinite number of unit cells. In computer simulations, one of these is the original simulation box, and others are copies called *images*.
- During the simulation, only the properties of the original simulation box need to be recorded and propagated.
- The *minimum-image convention* is a common form of PBC particle bookkeeping in which each individual particle in the simulation interacts with the closest image of the remaining particles in the system.
- One example of periodic boundary conditions can be defined according to smooth real functions.

$\phi : \mathbb{R}^n \rightarrow \mathbb{R}$  by

$$\begin{aligned} \frac{\partial^m}{\partial x_1^m} \phi(a_1, x_2, \dots, x_n) &= \frac{\partial^m}{\partial x_1^m} \phi(b_1, x_2, \dots, x_n), \\ \frac{\partial^m}{\partial x_2^m} \phi(x_1, a_2, \dots, x_n) &= \frac{\partial^m}{\partial x_2^m} \phi(x_1, b_2, \dots, x_n), \\ &\dots, \\ \frac{\partial^m}{\partial x_n^m} \phi(x_1, x_2, \dots, a_n) &= \frac{\partial^m}{\partial x_n^m} \phi(x_1, x_2, \dots, b_n) \end{aligned}$$

for all  $m = 0, 1, 2, \dots$  and for constants  $a_i$  and  $b_i$ .

### 9. Explain tangential boundary conditions.



$$\int_a^b \vec{E} \cdot d\vec{l} + \int_b^c \vec{E} \cdot d\vec{l} + \int_c^d \vec{E} \cdot d\vec{l} + \int_d^a \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_{abcd} \vec{B} \cdot d\vec{s}$$

Area enclosed is infinitely small. Therefore RHS  $\rightarrow 0$ .

"ad" and "cb" are infinitely small. Therefore,  $\int_a^b$  and  $\int_d^a \rightarrow 0$ .

$\vec{E}_2$  (inside conductor) is zero. Therefore,  $\int_c^d \rightarrow 0$ .

This leaves:

$$\int_a^b \vec{E} \cdot d\vec{l} = 0$$

If  $ab$  distance is finite, but small, then  $\vec{E}$  is constant, and we have:

$$E_t \cdot \Delta = 0 \quad E_t = \text{component of } \vec{E} \text{ along } ab; \\ \text{i.e., tangential component}$$

Thus,  $E_t = 0$ .

In vector form, this can be written:

$$\hat{e}_n \times \vec{E} = 0 \text{ . where } \hat{e}_n \text{ is unit normal.}$$

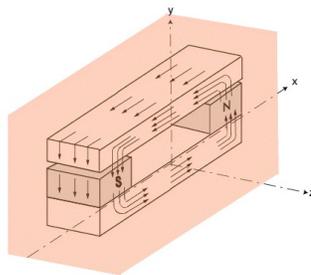
A very important corollary of this boundary condition is that, since  $E_t = 0$  at the surface of a perfect conductor,

$$\int_a^b \vec{E} \cdot d\vec{l} = 0$$

between any point on or inside the conductor. Therefore, **a perfect conductor is an equipotential surface.**

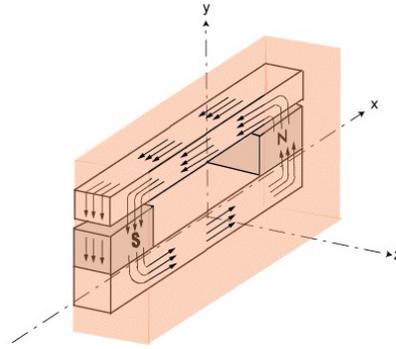
### 10.Explain flux normal condition.

- Magnetic flux boundary conditions impose constraints on the direction of the magnetic flux on a model boundary. This boundary condition may only be applied to faces. By default, this feature constrains the flux to be normal to all exterior faces.
- Selecting Flux Parallel forces the magnetic flux in a model to flow parallel to the selected face. In the figure below, the arrows indicate the direction of the magnetic flux. It can be seen that the flux flows parallel to the xy plane (for any z coordinate).

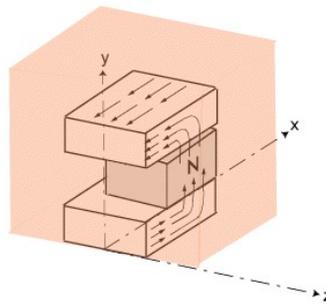


A flux parallel condition is required on at least one face of the simulation model. It is typically applied on the outer faces of the air body to contain the magnetic flux inside the simulation domain or on symmetry plane faces where the flux is known to flow parallel to the face. To set this feature, right-click on the Magnetostatic environment item in the tree and select Magnetic Flux Parallel from the Insert

context menu or click on the Magnetic Flux Parallel button in the toolbar. It can only be applied to geometry faces and Named Selections (faces).



Half-symmetry model of a kept magnet system. Note that the XY-plane is a Flux Parallel boundary. The flux arrows flow parallel to the plane.



Half-symmetry model of a kept magnet system. Note that the YZ-plane is a Flux Normal boundary. The flux arrows flow normal to the plane. This is a natural boundary condition and requires no specification.

## UNIT V DESIGN APPLICATIONS

### 1.Explain Electrical insulators.

Electrical Insulator must be used in electrical system to prevent unwanted flow of [current](#) to the earth from its supporting points. The insulator plays a vital role in electrical system. [Electrical Insulator](#) is a very high resistive path through which practically no current can flow. In transmission and distribution system, the [overhead conductors](#) are generally supported by supporting towers or poles. The towers and poles both are properly grounded. So there must be insulator between tower or pole body and current carrying conductors to prevent the flow of current from [conductor](#) to earth through the grounded supporting towers or poles.

### Insulating Material

The main cause of failure of overhead line insulator, is flash over, occurs in between line and earth during abnormal over [voltage](#) in the system. During this flash over, the huge heat produced by arcing, causes puncher in insulator body. Viewing this phenomenon the materials used for electrical insulator, has to posses some specific properties.

### **Properties of Insulating Material**

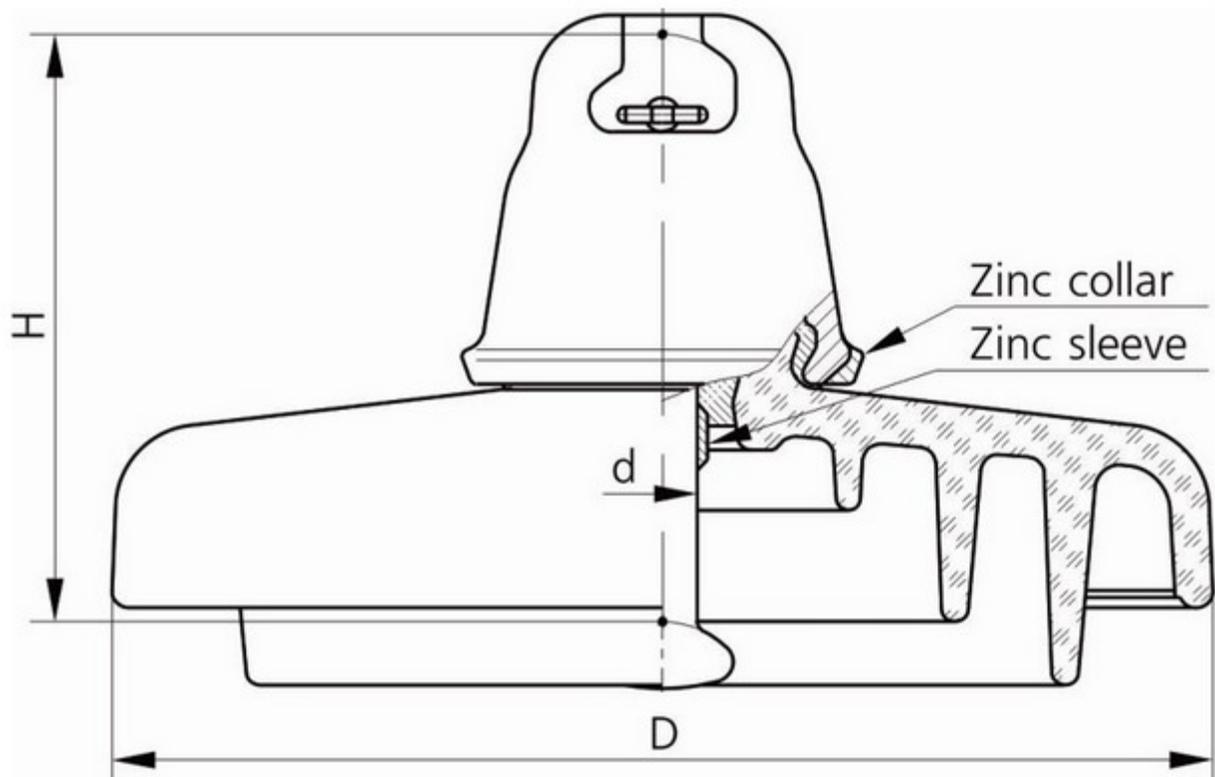
The materials generally used for insulating purpose is called insulating material. For successful utilization, this material should have some specific properties as listed below-

1. It must be mechanically strong enough to carry tension and weight of [conductors](#).
2. It must have very high dielectric strength to withstand the voltage stresses in High Voltage system.
3. It must possesses high Insulation [Resistance](#) to prevent leakage current to the earth.
4. The insulating material must be free from unwanted impurities.
5. It should not be porous.
6. There must not be any entrance on the surface of electrical insulator so that the moisture or gases can enter in it.
7. There physical as well as electrical properties must be less effected by changing temperature.

### **Insulators for HVDC :**

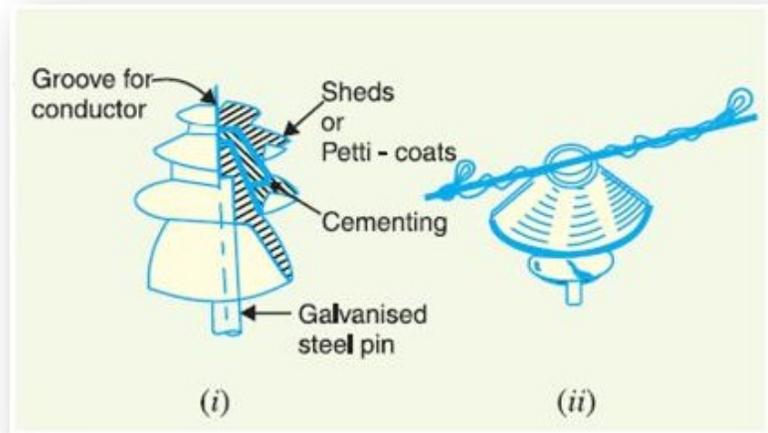
1. Special glass with low ion conductivity
2. High resistance properties of the glass at high temperatures
3. Adapted glass part design allows to avoid dump between ribs and insulator flashover
4. Special fixing device design protects it from galvochemical corrosion: - zinc sleeve bonded to the pin  
- zinc collar bonded to the cap

5. Metallurgical bond of the zinc collar and cast iron cap reaches up to 100%.
6. High resistance to the ionic migration within the glass part reaches a record values as a result of GIG know-how
7. All technical requirements and testing are in accordance with IEC standards.



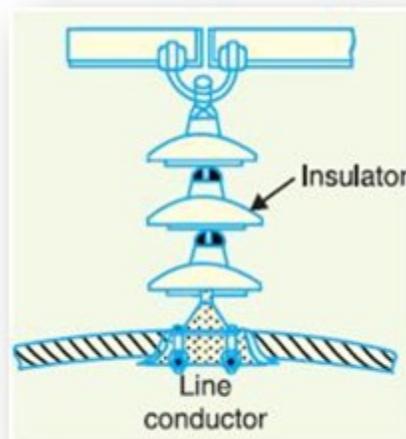
**2.Explain types of insulators.**

**Pin type Insulators**



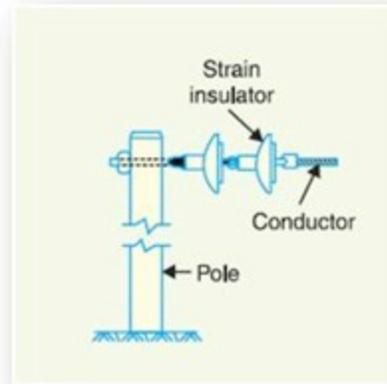
The pin type insulator is secured to the cross-arm on the pole. There is a groove on the upper end of the insulator for housing the conductor. The conductor passes through this groove and is bound by the annealed wire of the same material as the conductor. Pin type insulators are used for transmission and distribution of electric power at voltages up to 33 kV. Beyond operating voltage of 33 kV, the pin type insulators become too bulky and hence uneconomical.

### Suspension Type



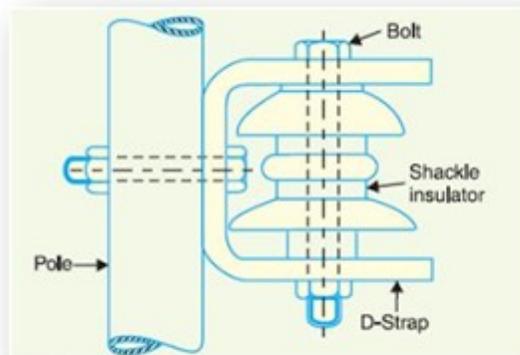
For high voltages ( $>33$  kV), it is a usual practice to use suspension type insulators shown in Figure. Consist of a number of porcelain discs connected in series by metal links in the form of a string. The conductor is suspended at the bottom end of this string while the other end of the string is secured to the cross-arm of the tower. Each unit or disc is designed for low voltage, say 11 kV. The number of discs in series would obviously depend upon the working voltage. For instance, if the working voltage is 66 kV, then six discs in series will be provided on the string.

## Strain Insulators



When there is a dead end of the line or there is corner or sharp curve, the line is subjected to greater tension. In order to relieve the line of excessive tension, strain insulators are used. For low voltage lines ( $< 11$  kV), shackle insulators are used as strain insulators. However, for high voltage transmission lines, strain insulator consists of an assembly of suspension insulators as shown in Figure. The discs of strain insulators are used in the vertical plane. When the tension in lines is exceedingly high, at long river spans, two or more strings are used in parallel.

## Shackle Insulators



The shackle insulators were used as strain insulators. But now a day, they are frequently used for low voltage distribution lines. Such insulators can be used either in a horizontal position or in a vertical position. They can be directly fixed to the pole with a bolt or to the cross arm.

### 3.Explain voltage stress in insulators.

- The HV insulators are located at the top of the utility poles. Partial discharge (PD) of the high voltage overhead insulator can be defined as local electric stress on the surface of the insulator or inside the insulation materials.
- An insulator is usually modelled as a coaxial cylinder system. The electric stress at any point in the insulation material is given by

$$E(x) = \frac{U}{x \ln \frac{r_o}{r_i}}$$

Here U= Applied voltage,  $r_i$ = Conductor radius,  $r_o$ = Sheath radius,  $x$ = radial distance to any point in the insulation.

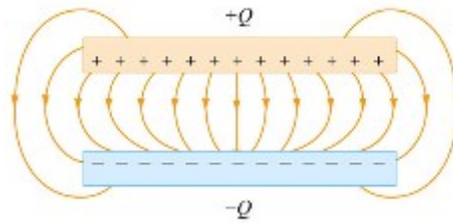
- It occurs on the surface of the conductor which is the most probable location for instantaneous failure. This is more likely to occur if serious insulation defects are near the conductor surface

$$E_{min} = \frac{U}{r_o \ln \frac{r_o}{r_i}} = \text{minimum stress.}$$

- The minimum stress occurs at the outer sheath and it becomes a critical factor if insulation defects are in the vicinity of the outer sheath.
- The mean stress (i.e.  $E_{mean}$ ) across the insulation is critical if insulation defects are uniformly located throughout the bulk of the material.

#### 4.Explain capacitance calculation.

Consider two metallic plates of equal area  $A$  separated by a distance  $d$ , as shown



To find the capacitance  $C$ , we first need to know the electric field between the plates. A real capacitor is finite in size. Thus, the electric field lines at the edge of the plates are not straight lines, and the field is not contained entirely between the plates. This is known as edge effects, and the non-uniform fields near the edge are called the fringing fields. edge effects, and the non-uniform fields near the edge are called the fringing fields. In the field lines are drawn by taking into consideration edge effects. However, in what follows, we shall ignore such effects and assume an idealized situation, where field lines between the plates are straight lines. In the limit where the plates are infinitely large, The system has planar symmetry and we can calculate the electric field every where using Gauss's law

$$\oiint_S \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

$$EA' = \frac{q_{\text{enc}}}{\epsilon_0} = \frac{\sigma A'}{\epsilon_0} \Rightarrow E = \frac{\sigma}{\epsilon_0}$$

$$\Delta V = V_- - V_+ = -\int_+^- \vec{E} \cdot d\vec{s} = -Ed$$

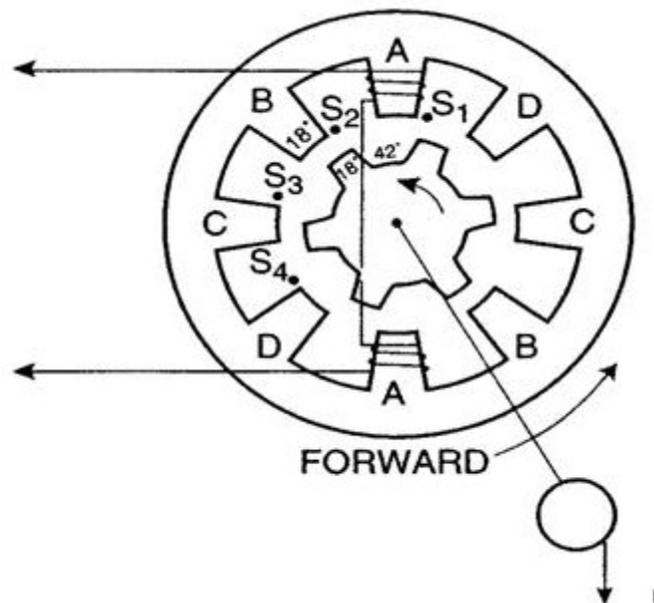
The potential difference between the plates is

$$C = \frac{Q}{|\Delta V|} = \frac{\epsilon_0 A}{d} \quad (\text{parallel plate})$$

## 5.Explain the construction and working of SRM.

The switched reluctance motor (SRM) is a type of a stepper motor, an electric motor that runs by reluctance torque. Unlike common DC motor types, power is delivered to windings in the stator (case) rather than the rotor.

- The structure of a switched reluctance motor is shown below. This is a 4-phase machine with 4 stator-pole pairs and 3 rotor-pole pairs (8/6 motor).
- The rotor has neither windings nor permanent magnets. The stator poles have concentrated winding rather than sinusoidal winding. E
- Each stator-pole pair winding is excited by a converter phase, until the corresponding rotor pole-pair is aligned and is then de-energized.
- The stator-pole pairs are sequentially excited using a rotor position encoder for timing.

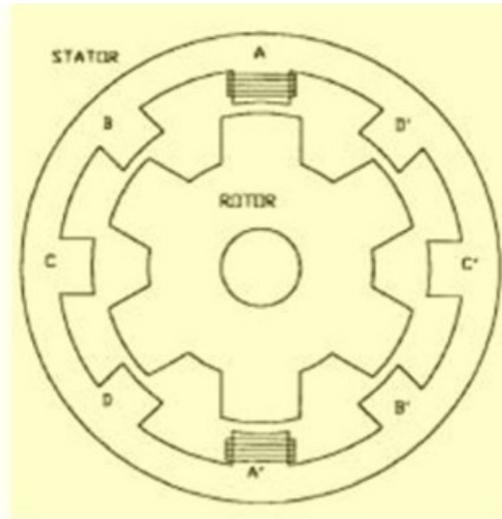


- The stator poles have concentrated winding rather than sinusoidal winding.
- Each stator-pole pair winding is excited by a converter phase, until the corresponding rotor pole-pair is aligned and is then de-energized.
- The stator-pole pairs are sequentially excited using a rotor position encoder for timing.

### CONSTRUCTION:

- Stator and rotor are salient in structure
- Stator windings are independent concentrated windings which are excited with switches from source
- No field windings hence singly excited-diametrically opposite armature windings are connected to form a phase for bidirectional control and self starting, num of rotor poles are less than num of stator poles

- Single stack and multi stack construction possible

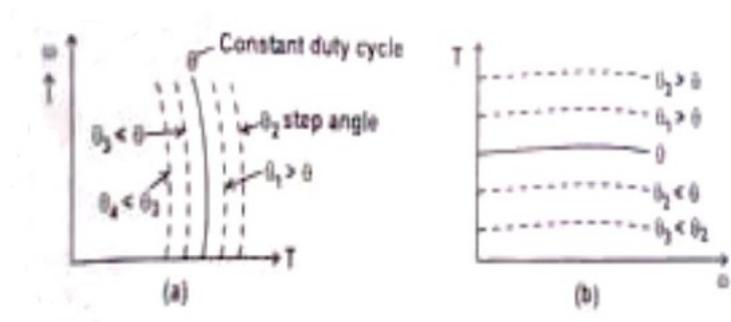


### WORKING

- When phase a is excited reluctance torque causes rotor to turn until it aligns with axis of phase a.
- Excitation is changed to b and a is deexcited before alignment rotation is in direction of energisation.
- Direction of rotation reversed by reversing sequence of excitation.
- Speed depends on magnitude of input microstepping can be done for single stack only 1 rotor and stator.
- For multi stack operation, number of rotor and stator depends on number of phases position of min reluctance changed with help of position sensors.
- When phase is excited after the rotor passes point of min reluctance, reverse torque acts [regenerative braking].

### **6.Explain torque speed characteristics of SRM.**

- Torque developed (i.e.) average torque developed but SRM depends upon the current wave form of SRM phase winding.
- Current waveform depends upon the conduction period and chopping details. It also depends upon the speed. Consider a case that conduction angle  $\Theta$  is constant and the chopper duty cycle is 1. (i.e.) it conducts continuously.
- For low speed operating condition, the current is assumed to be almost flat shaped. Therefore the developed torque is constant.
- For high speed operating condition, the current wave form gets changed and the average torque developed gets reduced.
- The following fig. represents the speed torque characteristics of SRM for constant  $\Theta$  and duty cycle. It is constant at low speeds and slightly droops as speed increases. For various other constant value of  $\Theta$ , the family of curves for the same duty cycle is shown in fig.

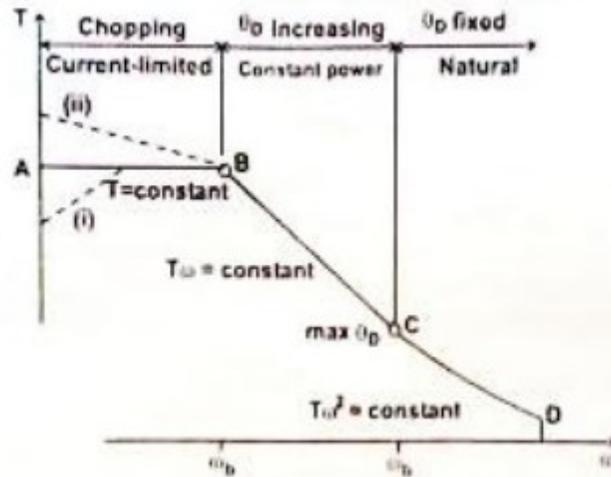


Torque speed characteristics at constant conduction angle  $\theta$  and duty cycle

Torque speed characteristics for fixed  $\theta$  and for various duty cycles are shown.  $\theta$  and duty cycle are varied by suitably operating the semiconductor devices.

### Torque Speed Capability Curve

Maximum torque developed in a motor and the maximum power that can be transferred are usually restricted by the mechanical subsystem design parameters. For given conduction angle the torque can be varied by varying the duty cycle of the chopper. However the maximum torque developed is restricted to definite value based on mechanical consideration.



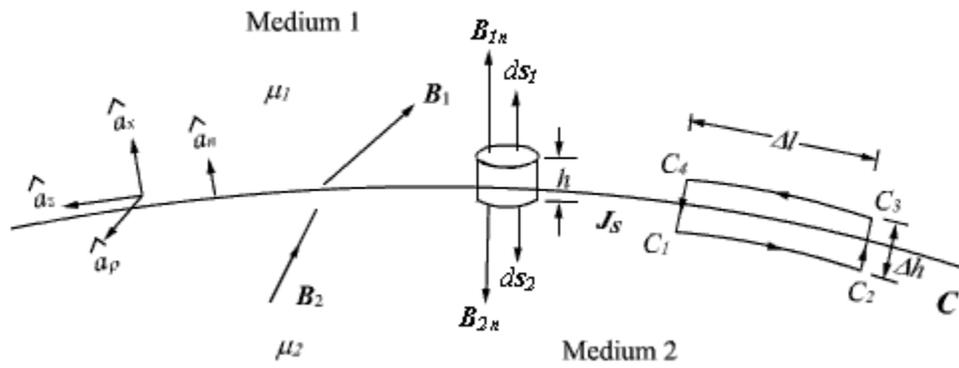
Torque speed characteristic of switched reluctance motor

- AB in the fig. represents constant maximum torque region of operation. At very low speeds, the torque / speed capability curve may deviate from the clock torque characteristics.
- If the chopping frequency is limited or if the bandwidth of the current regulator is limited, it is difficult to limit the current without the help of self emf of the motor and the current reference may have to be reduced.
- If very low windage and core loss permit the chopper losses to be increased, so that with higher current a higher torque is obtained.

- Under intermittent condition of course very much higher torque can be obtained in any part of the speed range up to  $\omega_b$ .
- The motor current limits the torque below base speed. The 'corner point' or base speed  $\omega_b$  is the highest speed at which maximum current can be supplied at rated voltage with fixed firing angles.
- If these angles are still kept fixed, the maximum torque at rated voltage decreases with speed squared. But if the conduction angle is increased, (i.e.)  $\theta$  is decreased, there is a considerable speed range over which maximum current can be still be forced into the motor.
- This maintains the torque at a higher level to maintain constant power characteristic. But the core losses and windage losses increases with the speed.
- Thus the curve BC represents the maximum permissible torque at each speed without exceeding the maximum permissible power transferred.
- This region is obtained by varying  $\theta$  to its maximum value  $\theta_{max}$ .  $\theta_{max}$  is dwell angle of the main switching devices in each phase. Point C corresponds to maximum permissible power; maximum permissible conduction angle
- $\theta_{max}$  and duty cycle of the chopper is unity.
- Curve CD represents  $T \omega^2$  constant. The conduction angle is kept maximum and duty cycle is maximum by maintaining  $T \omega^2$  constant. D corresponds to maximum  $\omega$  permissible.
- The region between the curve ABCD and X axis is the —permissible region of operation of SRM ||.

### 7.Explain boundary conditions of magnetic fields.

Similar to the boundary conditions in the electro static fields, here we will consider the behavior of  $\vec{B}$  and  $\vec{H}$  at the interface of two different media. In particular, we determine how the tangential and normal components of magnetic fields behave at the boundary of two regions having different permeabilities. The figure shows the interface between two media having permeabilities  $\mu_1$  and  $\mu_2$ ,  $\hat{a}_n$  being the normal vector from medium 2 to medium 1.



To determine the condition for the normal component of the flux density vector  $\vec{B}$ , we consider a small pill box P with vanishingly small thickness  $h$  and having an elementary area  $\Delta S$  for the faces. Over the pill box, we can write

$$\oint_S \vec{B} \cdot d\vec{S} = 0$$

Since  $h \rightarrow 0$ , we can neglect the flux through the sidewall of the pill box.

$$\therefore \int_{\Delta S} \vec{B}_1 \cdot d\vec{S}_1 + \int_{\Delta S} \vec{B}_2 \cdot d\vec{S}_2 = 0$$

$$d\vec{S}_1 = dS \hat{a}_n \text{ and } d\vec{S}_2 = dS \left( -\hat{a}_n \right)$$

$$\therefore \int_{\Delta S} B_{1n} dS - \int_{\Delta S} B_{2n} dS = 0$$

where

$$B_{1n} = \vec{B}_1 \cdot \hat{a}_n \text{ and } B_{2n} = \vec{B}_2 \cdot \hat{a}_n$$

Since  $\Delta S$  is small, we can write

$$(B_{1n} - B_{2n})\Delta S = 0$$

or,  $B_{1n} = B_{2n}$

That is, the normal component of the magnetic flux density vector is continuous across the interface.

In vector form,

$$\hat{a}_n \cdot (\vec{B}_1 - \vec{B}_2) = 0$$

To determine the condition for the tangential component for the magnetic field, we consider a closed path C as shown in figure 4.8. By applying Ampere's law we can write

$$\oint \vec{H} \cdot d\vec{l} = I$$

Since  $h \rightarrow 0$ ,

$$\int_{c_1-c_2} \vec{H} \cdot d\vec{l} + \int_{c_3-c_4} \vec{H} \cdot d\vec{l} = I$$

We have shown in figure 4.8, a set of three unit vectors  $\hat{a}_n$ ,  $\hat{a}_t$  and  $\hat{a}_p$  such that they satisfy  $\hat{a}_t = \hat{a}_p \times \hat{a}_n$

(R.H. rule). Here  $\hat{a}_t$  is tangential to the interface and  $\hat{a}_p$  is the vector perpendicular to the surface

enclosed by C at the interface.

The above equation can be written as

$$\vec{H}_1 \cdot \hat{a}_t \Delta l - \vec{H}_2 \cdot \hat{a}_t \Delta l = I = J_s \Delta l$$

or,

$$H_{1t} - H_{2t} = J_s$$

i.e., tangential component of magnetic field component is discontinuous across the interface where a free surface current exists. If  $J_s = 0$ , the tangential magnetic field is also continuous. If one of the medium is a perfect conductor  $J_s$  exists on the surface of the perfect conductor.

In vector form we can write,

$$\begin{aligned} & (\vec{H}_1 - \vec{H}_2) \cdot \hat{a}_t \Delta l \\ &= (\vec{H}_1 - \vec{H}_2) \cdot (\hat{a}_p \times \hat{a}_n) \Delta l \\ &= J_s \Delta l = \vec{J}_s \cdot \hat{a}_p \Delta l \end{aligned}$$

Therefore,

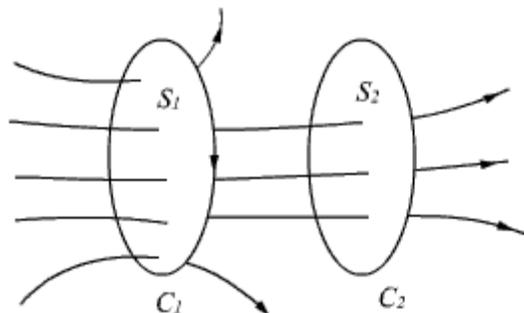
$$\hat{a}_n \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$$

## 8. Discuss inductance calculations.

Resistance, capacitance and inductance are the three familiar parameters from circuit theory. We have already discussed about the parameters resistance and capacitance in the earlier chapters. In this section, we discuss about the parameter inductance. let us first introduce the concept of flux linkage. If in a coil with  $N$  closely wound turns around where a current  $I$  produces a flux  $\phi$  and this flux links or encircles each of the  $N$  turns, the flux linkage  $\Lambda$  is defined as  $\Lambda = N\phi$ . In a linear medium, where the flux is proportional to the current, we define the self inductance  $L$  as the ratio of the total flux linkage to the current which they link.

i.e., 
$$L = \frac{\Lambda}{I} = \frac{N\phi}{I}$$

To further illustrate the concept of inductance, let us consider two closed loops  $C_1$  and  $C_2$  as shown in the figure,  $S_1$  and  $S_2$  are respectively the areas of  $C_1$  and  $C_2$ .



If a current  $I_1$  flows in  $C_1$ , the magnetic flux  $B_1$  will be created part of which will be linked to  $C_2$  as

shown in Figure.

$$\phi_{12} = \int_{S_2} \vec{B}_1 \cdot d\vec{S}_2$$

In a linear medium,  $\phi_{12}$  is proportional to  $I_1$ . Therefore, we can write

$$\phi_{12} = L_{12} I_1$$

where  $L_{12}$  is the mutual inductance. For a more general case, if  $C_2$  has  $N_2$  turns then

$$\Lambda_{12} = N_2 \phi_{12}$$

$$\text{and } \Lambda_{12} = L_{12} I_1$$

$$\text{or } L_{12} = \frac{\Lambda_{12}}{I_1}$$

i.e., the mutual inductance can be defined as the ratio of the total flux linkage of the second circuit to the current flowing in the first circuit. The magnetic flux produced in  $C_1$  gets linked to itself and if  $C_1$  has  $N_1$  turns further  $\Lambda_{11} = N_1 \phi_{11}$ , where  $\phi_{11}$  is the flux linkage per turn.

Therefore, self inductance

$$L_{11} (\text{or } L \text{ as defined earlier}) = \frac{\Lambda_{11}}{I_1}$$

As some of the flux produced by  $I_1$  links only to  $C_1$  & not  $C_2$ .

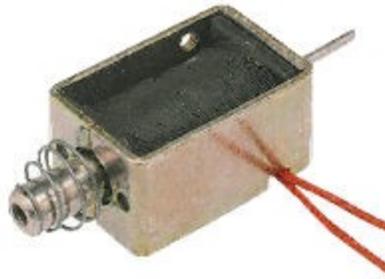
$$\Lambda_{11} = N_1 \phi_{11} > N_2 \phi_{12} = \Lambda_{12}$$

$$L_{12} = \frac{d\Lambda_{12}}{dI_1} \quad \text{and} \quad L_{11} = \frac{d\Lambda_{11}}{dI_1}$$

Further in general, in a linear medium,

## 9. Describe about solenoid actuators.

The solenoid actuators work on the same basic principle as the electromechanical relay seen in the previous tutorial and just like relays, they can also be switched and controlled using transistors or MOSFET's. A "Linear Solenoid" is an electromagnetic device that converts electrical energy into a mechanical pushing or pulling force or motion.



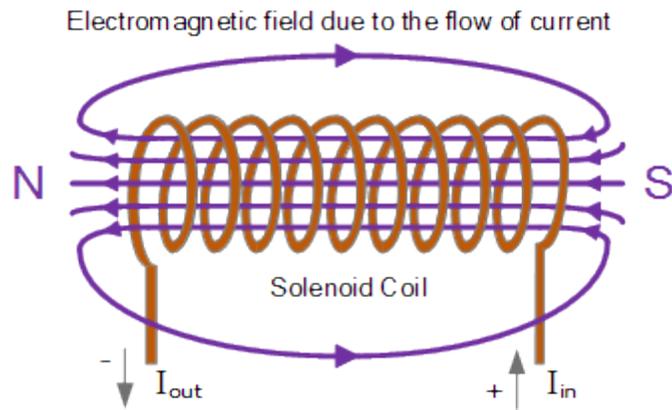
Solenoids basically consist of an electrical coil wound around a cylindrical tube with a ferro-magnetic actuator or "plunger" that is free to move or slide "IN" and "OUT" of the coil's body. Solenoids can be used to electrically open doors and latches, open or close valves, move and operate robotic limbs and mechanisms, and even actuate electrical switches just by energising its coil.

*Solenoids* are available in a variety of formats with the more common types being the *linear solenoid* also known as the linear electromechanical actuator, (LEMA) and the *rotary solenoid*. Both types of solenoid, linear and rotational are available as either a holding (continuously energised) or as a latching type (ON-OFF pulse) with the latching types being used in either energised or power-off applications. Linear solenoids can also be designed for proportional motion control where the plunger position is proportional to the power input.

When electrical current flows through a conductor it generates a magnetic field, and the direction of this magnetic field with regards to its North and South Poles is determined by the direction of the current flow within the wire. This coil of wire becomes an "**Electromagnet**" with its own north and south poles exactly the same as that for a permanent type magnet.

The strength of this magnetic field can be increased or decreased by either controlling the amount of current flowing through the coil or by changing the number of turns or loops that the coil has. An example of an "Electromagnet" is given below.

### **Magnetic Field produced by a Coil**

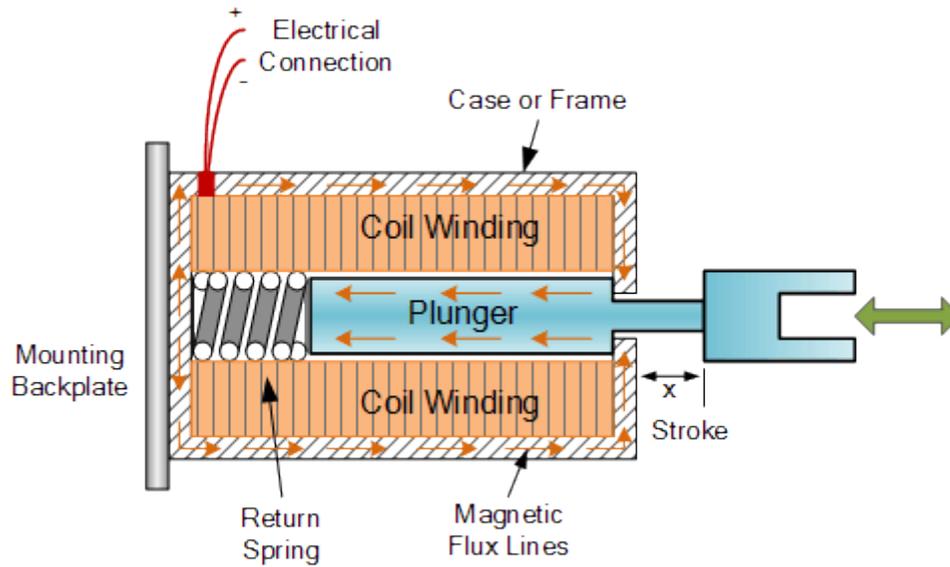


When an electrical current is passed through the coils windings, it behaves like an electromagnet and the plunger, which is located inside the coil, is attracted towards the centre of the coil by the magnetic flux setup within the coils body, which in turn compresses a small spring attached to one end of the plunger. The force and speed of the plungers movement is determined by the strength of the magnetic flux generated within the coil.

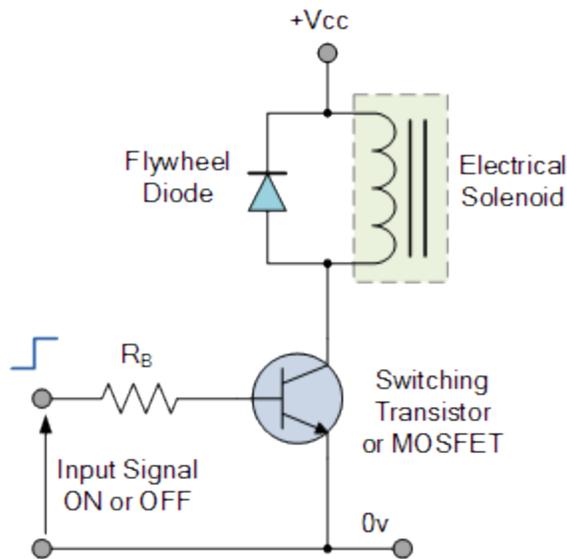
When the supply current is turned “OFF” (de-energised) the electromagnetic field generated previously by the coil collapses and the energy stored in the compressed spring forces the plunger back out to its original rest position. This back and forth movement of the plunger is known as the solenoids “Stroke”, in other words the maximum distance the plunger can travel in either an “IN” or an “OUT” direction, for example, 0 – 30mm.

### SOLENOID CONSTRUCTION

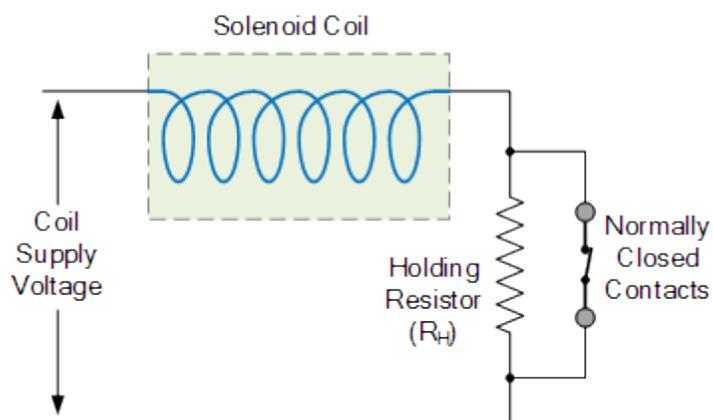
This type of solenoid is generally called a **Linear Solenoid** due to the linear directional movement and action of the plunger. Linear solenoids are available in two basic configurations called a “Pull-type” as it pulls the connected load towards itself when energised, and the “Push-type” that act in the opposite direction pushing it away from itself when energised. Both push and pull types are generally constructed the same with the difference being in the location of the return spring and design of the plunger.



### Switching Solenoids using a Transistor



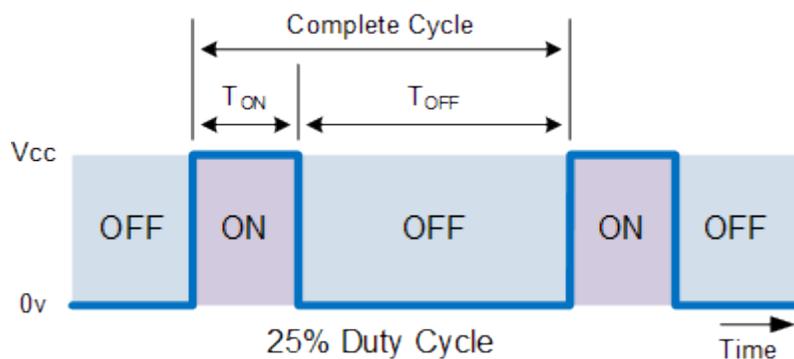
### Reducing Solenoid Energy Consumption



Here, the switch contacts are closed shorting out the resistance and passing the full supply current directly to the solenoid coils windings. Once energised the contacts which can be mechanically connected to the solenoids plunger action open connecting the holding resistor,  $R_H$  in series with the solenoids coil. This effectively connects the resistor in series with the coil.

The Duty Cycle (%ED) of a solenoid is the portion of the “ON” time that a solenoid is energised and is the ratio of the “ON” time to the total “ON” and “OFF” time for one complete cycle of operation. In other words, the cycle time equals the switched-ON time plus the switched-OFF time. Duty cycle is expressed as a percentage, for example:

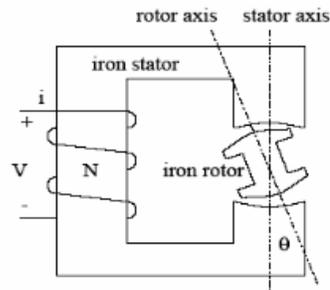
$$\text{Duty Cycle} = \frac{\text{"ON" time}}{\text{"ON" time} + \text{"OFF" time}} \times 100\%$$



Then if a solenoid is switched “ON” or energised for 30 seconds and then switched “OFF” for 90 seconds before being re-energised again, one complete cycle, the total “ON/OFF” cycle time would be 120 seconds, (30+90) so the solenoids duty cycle would be calculated as 30/120 secs or 25%. This means that you can determine the solenoids maximum switch-ON time if you know the values of duty cycle and switch-OFF time.

### 10. Develop the torque equation for the switched reluctance motor

Let us consider an elementary reluctance machine. The machine is single phase excited; that is, it carries only one winding on the stator. The excited winding is wound on the stator and the rotor is free to rotate.



$$\lambda(\theta) = L(\theta)i$$

The flux linkage is

where  $i$  is the independent input variable, i.e. the current flow through the stator. The general torque expression is given by

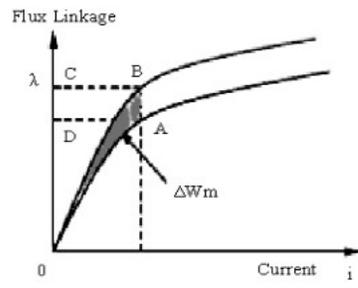
$$T_e = \left[ \frac{\partial W'}{\partial \theta} \right]_{i=\text{constant}}$$

where  $W'$  is the co-energy which is varying with respect to position of the motor. At any position the co-energy is the area below the magnetization curve as shown. The definite integral is given by

$$W' = \int_0^i \lambda(\theta, i) di$$

Where  $\lambda(\theta, i)$  is the flux linkage with respect to angular position  $\theta$  and current ' $i$ '. So the torque equation becomes

$$T_e = \int_0^i \frac{\partial \lambda(\theta, i)}{\partial \theta} di$$



$$T_e = \sum_{j=1}^m T_{ej}$$