DHANALAKSHMI SRINIVASAN ENGINEERING COLLEGE
PERAMBALUR 621212
DEPARTMENT OF MATHEMATICS
SUB CODE/ TITLE:MA8353- TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATION
QUESTION BANK
(BE., AERO SPACE & MECHANICAL - III SEMESTER)
UNIT-I (PARTIAL DIFFERENTIAL EQUATION)
PART-A

1. Form the partial differential equation by eliminating a, b from $z = (x^2 + a^2)(y^2 + b^2)$.

2. Form the PDE by eliminating the constants a and b form i)$z = ax^n + by^n$

ii) $4(1 + a^2)z = (x + ay + b)^2$

3. Find the partial differential equation of all planes passing through the origin.

4. Obtain partial differential equation by eliminating arbitrary constants a and b from

$(x-a)^2 + (y-b)^2 + z^2 = 1$

5. Find the PDE of all planes having equal intercepts on the x and y axis.

6. Eliminate the arbitrary function f from z= $f\left(\frac{y}{x}\right)$ **and form the PDE.**

7. Form partial differential equation by eliminating arbitrary function $z = f(xy)$.

8. Form the PDE by eliminating the arbitrary function from $\phi\left[z^2 - xy, \frac{x}{z}\right] = 0$

9. Find the complete integral of $p + q = 1$ **where** $p = \frac{\partial z}{\partial x}$ **and** $q = \frac{\partial z}{\partial y}$.

10. Write down the complete solution of $z = px + qy + c\sqrt{1 + p^2 + q^2}$ **.**

11 Find the singular integral of the partial differential equation $z = px + qy + p^2 - q^2$.

12. Find the solution of $px^2 + qy^2 = z^2$

13.Find the general solution of $5\frac{\partial^2 z}{\partial x^2} - 12\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$.

14. Solve $(D^2 - DD' + D' - 1)z = 0$

15. Solve $(D^3 - 3DD'^2 + 2D'^3)z = 0$

16. Solve $(D^3 - 4D^2D' + 4DD'^2)\ Z = 0$.

17. Find the particular integral of $(D^2 - 2DD'^2 + D'^2)z = e^{x-y}$.

18. Find the particular integral of $(D^3 - 3D^2D' - 4DD'^2 + 12D'^3)z = sin(x + 2y)$.

19. Solve:$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial x} = 0$

20. Solve:$(D^4 - D'^4)Z = 0$

PART - B

1. a) Form the PDE by eliminating the arbitrary functions $\varphi(x^2+y^2+z^2, ax+by+cz)$.

   b) Solve $x(y-z)p + y(z-x)q = z(x-y)$.

2. a) Form the partial differential equation by eliminating f and ϕ from $z = f(y) + \phi(x+y+z)$.

   b) Solve $(D^2 - DD' - 2D'^2)Z = 2x + 3y + e^{2x+4y}$

3. a) Form the PDE by eliminating arbitrary function f and g from $z = x^2 f(y) + y^2 g(x)$

   b) Solve $z = px + qy + \sqrt{1+p^2+q^2}$.

4. a) Find the singular integral of the partial differential equation $z = px + qy + p^2 - q^2$

   b) Solve $(2D^2 - DD' - D'^2 + 6D + 3D'^3)z = xe^y$.

5. a) Solve $(\frac{p}{2}+x)^2 + \left(\frac{q}{2}+y\right)^2 = 1$ b) Solve $x^2p + y^2q = z(x+y)$

6. a) Form the PDE by eliminating arbitrary function from the relation $Z = y^2 + 2f(\frac{1}{x} + logy)$

   b) Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$..

7. a) Solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$

   b) Solve $(D^3 + D^2D' - 4DD'^2 - 4D'^3)z = cos(x+y)$

8. a) a) Solve $(D^2 - DD' - 20D'^2)z = e^{5x+y} + sin(4x-y)$.

   b) Solve $(x-2z)p + (2z-y)q = y - x$

9. a) Solve $(D^2 - DD' - 30D'^2)z = xy + e^{6x+y}$

   b)i) Find the general solution of $(3z - 4y)p + (4x - 2z)q = 2y - 3x$

   ii)Solve$(D^3 - 7DD'^2 - 6D'^3)z = sin(x+2y) + e^{2x+y}$

10. a) Solve $(D^2 - 2DD')z = e^{2x} + x^3y$.

    b) i) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$

    ii) Form the PDE by eliminating the arbitrary functions f and g in $z = xf(x+t) + g(x+t)$.

UNIT II FOURIER SERIES

PART-A

1. Find the constant term in the Fourier series corresponding to f(x) = $\cos^2 x$ expressed in the interval $(-\pi, \pi)$

2. If f(x) = $x^2 + x$ is expressed as a Fourier series in the interval (-2, 2) to which value this series converges at x=2

3. If the Fourier series corresponding to f(x) = x in the interval (0, 2π) is $\frac{a_0}{2}+\sum_{1}^{\infty}(a_n \cos nx + b_n \sin nx)$, without finding a_0, a_n, b_n find the value of $\frac{a_0^2}{2}+\sum_{1}^{\infty}(a_n^2+b_n^2)$.

4. State Dirichlet's condition for a given function to expand in Fourier series.

5. If $(\pi - x)^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{\cos nx}{n^2}$ in $0 < x < 2\pi$ then deduce the value of $\sum_{n=1}^{\infty}\frac{1}{n^2}$

6. Find a Fourier sine series for the functionf(x) = 1 , i) 0<x<π ii) (0,2)

7. If the Fourier series for the function f(x) = $\begin{cases} 0 & 0 < x < \pi \\ \sin x & \pi < x < 2\pi \end{cases}$ is

f (x)=$-\frac{1}{\pi}+\frac{2}{\pi}\left[\frac{\cos 2x}{1.3}+\frac{\cos 4x}{3.5}+\frac{\cos 6x}{5.7}+....\right]+\frac{\sin x}{2}$ deduce $\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-..........\infty=\frac{\pi-2}{4}$

8. What is the constant term a_0 and the coefficient of cosnx, a_n in the Fourier series expansion of f(x) = x - x^3 in $(-\pi, \pi)$.

9. State Parseval's identity for full range expansion of f(x) as Fourier series in (0,2l)

10. Find the Fourier constants b_n for xsinx in $(-\pi, \pi)$

11. What do you mean by Harmonic analysis.

12. In the Fourier expansion of f(x)= $\begin{cases} 1+\frac{2x}{\pi}, -\pi < x < 0 \\ 1-\frac{2x}{\pi}, 0 < x < \pi \end{cases}$ in $(-\pi, \pi)$, find b_n.

13. Find b_n in the expansion of x^2 as a Fourier series in $(-\pi, \pi)$

14. If f(x) is an odd function defined in $(-l, l)$, what are the values of a_0 and a_n

15. State Parseval's Theorem on Fourier series

16. Does f(x) = tanx possess a Fourier expansion

17. Find a_n in expanding e^{-x} as Fourier series in $(-\pi, \pi)$

18. State Parseval's identity for the half-range cosine expansion of f(x) in (0, 1).

19. Find the root mean square value of the function f(x) = x in (0, l).

20. Find the value of a_0 in the Fourier series expansion of $f(x) = e^x in\ (0, 2\pi)$

Part-B

1. **a) Find the Fourier series of the function $f(x) = (\pi - x)^2$, in $(0, 2\pi)$ with periodicity 2π.**
   b) Find the Fourier series up to second Harmonic for the following data for y with period 6

x	0	1	2	3	4	5
y	9	18	24	28	26	20

2. **a) Obtain the Fourier series for f(x) = 1 + x + x^2 in ($-\pi$, π).**
   b) Find the half range sine series of $f(x) = x \cos x$ in $(0, \pi)$

3. **a) Find the Fourier series for f(x) = $|\cos x|$ in the interval ($-\pi$, π)**
   b) Obtain the cosine series for f(x) = x in 0 < x < π and deduce that $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^4}=\frac{\pi^4}{96}$

4. a) Obtain the Fourier series for the function $f(x) = \begin{cases} \pi x & 0 \le x \le 1 \\ \pi(2-x) & 1 \le x \le 2 \end{cases}$

b) The following table gives the variations of a periodic current over a period T

X	0	T/6	T/3	T/2	2T/3	5T/6	T
f(x)	1.98	1.30	1.05	1.30	-.088	-0.25	1.98

5. a) Find the Fourier series expansion of the periodic function f(x) of period $2l$ **define by**

f(x) = $l+x \quad -l \le x \le 0$

$= l-x \quad 0 \le x \le l$ **Deduce that** $\sum_{1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$

b) Obtain the half range cosine series for f(x)= $(x-1)^2$ $\boldsymbol{in}$ $0 < x < 1$. **Deduce that** $\sum_{1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$

6. a) Find the complex form of Fourier series of $e^{-ax}, -l < x < l$. **Deduce that when a is constant other than an integer** $\cos ax = \sin al \sum_{n=-\infty}^{\infty} \frac{al}{a^2l^2 - n^2\pi^2} (-1)^n e^{in\pi x/l}$

b) Find the Fourier series expansion of $f(x) = \begin{cases} -x+1 \ \boldsymbol{for} -\pi < x < 0 \\ x+1 \ \boldsymbol{for} \ 0 < x < \pi \end{cases}$

7. a) Expand f(x)= $\begin{cases} 1 + \frac{2x}{\pi}, -\pi < x < 0 \\ 1 - \frac{2x}{\pi}, 0 < x < \pi \end{cases}$ **as a full range Fourier series in the interval** $(-\pi, \pi)$**.Hence deduce that** $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\dots = \frac{\pi^2}{8}$.

b) Obtain Sine series for f(x) = x $\quad 0 \le x \le \frac{l}{2}$

$= l - x \quad \frac{l}{2} \le x \le l$

8. a) Find half range cosine series given $f(x) = x \quad 0 \le x \le 1$

$= 2 - x \quad 1 \le x \le 2$

b)Calculate the first 3 harmonics of the Fourier of f(x) from the following data

x	0	30	60	90	120	150	180	210	240	270	300	330
f(x)	1.8	1.1	0.3	0.16	0.5	1.3	2.16	1.25	1.3	1.52	1.76	2.0

9. a) Obtain Fourier series for f(x) of period $2l$ **and defined as follows**

f(x)= $\begin{cases} l-x & in \quad 0 \le x \le l \\ 0 & in \quad l \le x \le 2l \end{cases}$ **Hence deduce that (i) 1-** $\frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots\dots = \frac{\pi}{4}$ **(ii)** $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\dots = \frac{\pi^2}{8}$

b)Find the half range sine series of f(x) = $4x - x^2$ **in the interval (0, 4).Hence deduce the value of the series**

$$\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots$$

10a) Find the Fourier series for f(x) = x^2 **in** $(-\pi, \pi)$**. Hence find**

i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots\dots\dots$ **ii)** $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots$ **.iii)** $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} \dots\dots\infty$

b) Find the Fourier series of $f(x) = x \sin x \ in \ 0 < x < 2\pi$**.Hence deduce the sum of the series**

$$\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-.........\infty=\frac{\pi-2}{4}$$

UNIT –III (APPLICATION OF PARTIAL DIFFERENTAIL EQUATIONS)

PART – A

1. **In the wave equation $\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$ what does c^2stand for?**
2. **In the diffusion equation $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$,what does α^2 stand for**
3. **What are all the solutions of one dimensional wave equation?**
4. **What is the basic difference between the solution of one dimensional wave equation and one dimensional heat equation?**
5. **Define steady state condition on heat flow**
6. **What are all the solutions of one dimensional heat equation?**
7. **In steady state conditions derive the solution of one dimensional heat equation.**
8. **State the assumptions in deriving the one dimensional equation(unsteady state)**
9. **A rod 30 cm long has its ends A and B kept at 20^0 C and 80^0 C respectively until steady state conditions prevail. Find the steady state temperature in the rod.**
10. **Write down the two dimensional heat flow equation in steady state and the possible solutions in steady state.**
11. **Write any two solutions of Laplace equation $u_{xx}+u_{yy}=0$ involving exponential terms in $x\ and\ y$.**
12. **Write down the boundary and initial conditions for the transverse vibrations of the string of length l with fixed ends with initial displacement $y=f(x)$.**
13. **If the string of length l are fixed and the mid point of the string is drawn aside through a height h and the string is released from rest, state the initial and boundary conditions.**
14. **A tightly stretched string of length $2l$ has its ends fastened at x = 0 and x =$2l$. The midpoint of the string is then taken to a height b and then released from rest in that position. Write the initial and boundary conditions.**
15. **A tightly stretched string with end points $x=0\ and\ x=l$ is initially at rest in equilibrium position. If it is set vibrating giving each point velocity $\lambda x(l-x)$. Write the initial and boundary conditions.**
16. **The ends A and B of a rod of length 10 cm long have their temperature kept 20^0 C and 70^0 C.find the steady state temperature on the rod.**
17. **An infinitely long plane uniform plate is bounded by two parallel edges and an end at right angles to them. The breadth is π, this edge $u(x,0)$ is maintained at a temperature 60^0C at all points & the other edges are at zero temperature. Write down the boundary conditions for finding the steady-state temperature.**
18. **A square plate is bounded by the lines x = 0, y = 0, x = 20, y = 20. Its faces are insulated. The temperature along the upper horizontal edge is given by u(x, 0) = x (20-x) in$0<x<20$. Write the boundary conditions.**
19. **A rectangular plate with insulated surface is 10cm wide and so long compared to its width that it may be considered infinite in length without introducing appreciable error. The**

temperature at short edge y = 0 is given by $u = \begin{cases} 20x & , \quad 0 \le x \le 5 \\ 20(10-x), & 5 \le x \le 10 \end{cases}$. And all the other three edges are kept at 0^0 C. write the boundary conditions.

20. Classify the PDE $(1-x^2)Z_{xx} - 2xyZ_{xy} + (1-y^2)Z_{yy} + xZ_x + 3x^2yz - 2Z = 0$

PART –B

1. A tightly stretched string of length l has its end fastened at x = 0, x = l. At $t = 0$,the string is in the form $y(x,0) = kx(l-x)$ and then released .Find the displacement of any point on the string at a distance of x from one end at time t>0.
2. A tightly stretched string of length $2l$ has its ends fastened at x = 0 and x = $2l$. The midpoint of the string is then taken to a height b and then released from rest in that position. Find the lateral displacement of a point of the string at time t from the instant of release.
3. A slightly stretched string of length l has its ends fastened at x = 0 and x = l is initially in a position given by $y(x,0) = y_0 sin^3 \frac{\pi x}{l}$.If it is released from rest from this position, find the displacement y at any distance x from one end and at any time.
4. If a string of length l is initially at rest in its equilibrium position and each of its points is given the velocity $v_0 \sin^3 \frac{\pi x}{l}$, determine the displacement of a point distant x from one end at time t.
5. A sting of length l is initially at rest in its equilibrium position and motion is started by giving each of its points a velocity given by $v = \begin{cases} cx & if\, 0 \le x \le \frac{l}{2} \\ c(l-x) & if\, \frac{l}{2} \le x \le l \end{cases}$. Find the displacement function y(x, t).
6. A rod 30 cm long has its ends A and B kept at 20^0 and 80^0 respectively until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0^0C and kept so. Find the resulting temperature function u(x, t) taking x = 0 at A.
7. The ends A and B of a rod l cm long have the temperature 40^0 and 90^0 respectively until steady state conditions prevail. The temperature at A is suddenly raised to 90^0 and at the same time that at B is lowered to 40^0. Find the temperature distribution in the rod at time t.
8. Find the solution of the equation $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ that satisfies the conditions $u(0,t) = 0$, $u(l,t) = 0$, for $t > 0$, and $u(x,0) = \begin{cases} x, 0 \le x \le l/2 \\ l - x, l/2 < x < l \end{cases}$
9. A square plate is bounded by the lines $x = 0, y = 0, x = 20, y = 20$. Its faces are insulated. The temperature along the upper horizontal edge is given by $u(x,0) = x(20-x)$ when $0 < x < 20$ while the other three edges are kept at 0^0 C. Find the steady state temperature in the plate.
10. A rectangular plate with insulated surface is 10cm wide. The two long edges and one short edge are kept at 0^0 C, while the temperature at short edge x = 0 is given by $u = \begin{cases} 20y & , \quad 0 \le y \le 5 \\ 20(10-y), & 5 \le y \le 10 \end{cases}$ Find the steady state temperature at any point in the plate.

UNIT- IV (FOURIER TRANSFORMS)

PART-A

1. State the Fourier integral theorem.

2. If $F_C(s)$ is the Fourier Cosine transform of $f(x)$, prove that the Fourier cosine transform of $f(ax)$ is $\frac{1}{a}F_c\left(\frac{s}{a}\right)$

3. If $F(s)$ is the Fourier transform of $f(x)$, write the formula for the Fourier of $f(x)= cos(ax)$ in term of F.

4. State the convolution theorem for Fourier transforms.

5. If F(s) is the Fourier transform of f(x), find the Fourier transform of $f(x-a)$.

6. If $F_s(s)$ is the Fourier sine transform of f(x), show that $F_s\{f(x)\cos ax\}=\frac{1}{2}[F_s(s+a)+F_s(s-a)]$

7. Find the Fourier transform of f(x) if $f(x)=\begin{cases}1, & |x|<a\\ 0, & |x|>a>0\end{cases}$

8. Find Fourier Cosine transform of e^{-x}

9. Find: $F\{x^n f(x)\}$

10. $F\left\{\frac{d^n f(x)}{dx^n}\right\}$ in terms of F.Tof f(x)

11. Write the Fourier transform pair.

12. Find Fourier sine transform of $\frac{1}{x}$.

13. State the Fourier transforms of the derivatives of a function.

14. Find the Fourier transform of $e^{-\alpha|x|}$, $\alpha>0$

15.Find the function f(x) whose sine transform is $\frac{e^{-as}}{s}$.

16. If F {f(x)} = $\overline{f}(s)$ then give the value of F {f (ax)}

17. Find the Fourier sine transform of $f(x)=e^{-x/2}$

18. State Modulation theorem on Fourier transforms.

19. Define self reciprocal with respect to Fourier transform

20. Prove that $F[f(x-a)] = e^{ias} F[f(x)]$

PART-B

1. a) Find the Fourier transform of $f(x)=\begin{cases}1 & \text{for } |x|<1\\ 0 & \text{otherwise}\end{cases}$, **Hence prove that** $\int_0^\infty \frac{\sin x}{x}dx=\int_0^\infty \frac{\sin^2 x}{x^2}dx=\frac{\pi}{2}$

b) Find the Fourier sine and cosine transform of $f(x)=\begin{cases}x, & 0<x<1\\ 2-x, & 1<x\le 2\\ 0, & x>2\end{cases}$.

2. a) Find the Fourier sine transform of $xe^{-\frac{x^2}{2}}$

b) Find the Fourier transform of $e^{-a^2x^2}$, $a > 0$. Hence show that $e^{-x^2/2}$ is self reciprocal under the Fourier transform

3. a) Find the Fourier transform of the function f(x) defined by

$$f(x) = \begin{cases} 1-x^2 & |x| \le 1 \\ 0 & |x| > 1 \end{cases}$$ **Hence prove that** (i) $\int_0^\infty \left\{\frac{\sin s - s\cos s}{s^3}\right\}\cos\frac{s}{2}\,ds = \frac{3\pi}{16}$

(ii) $\int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right)^2 ds = \frac{\pi}{15}$

b) Using Parseval's identity for Fourier Cosine transform of e^{-ax} **and evaluate** $\int_0^\infty \frac{dx}{(a^2+x^2)^2}$

4. a) Find the Fourier transform of $e^{-a|x|}$ if $a > 0$. **Deduce that** $\int_0^\infty \frac{1}{(x^2+a^2)^2}dx = \frac{\pi}{4a^3}$ if $a > 0$

b) Find the Fourier sine transform of $f(x) = \begin{cases} \sin x, & 0 < x < \pi \\ 0 & \pi \le x < \infty \end{cases}$

5. a) Find the Fourier cosine transform of e^{-4x}. Deduce that $\int_0^\infty \frac{\cos 2x}{x^2+16}dx = \frac{\pi}{8}e^{-8}$ **and** $\int_0^\infty \frac{x\sin 2x}{x^2+16}dx = \frac{\pi}{2}e^{-8}$

b) Using Parseval's identity evaluate $\int_0^\infty \frac{x^2 dx}{(x^2+a^2)^2}$**, if a>0**

6. a) Find the Fourier Transform of $e^{-a|x|}, a>0$. **Hence deduce that** $F\left[xe^{-a|x|}\right] = i\sqrt{\frac{2}{\pi}}\frac{2as}{\left(a^2+s^2\right)^2}$.

b) Evaluate $\int_0^\infty \frac{dx}{(x^2+16)(x^2+25)}$ **by transforms**

7. a) Find the Fourier transform of $f(x) = \begin{cases} 1-|x|, & |x| < 1 \\ 0, & otherwise \end{cases}$ **Hence deduce** $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$

b) Find Fourier Sine and Cosine transform of x^{n-1}**.Hence prove that** $\frac{1}{\sqrt{x}}$ **is self reciprocal under both cosine and sine transform**

8. a) Verify Parseval's Theorem of Fourier transform for the function $f(x) = \begin{cases} 0, & x<0 \\ e^{-x}, & x>0 \end{cases}$

b) Find the Fourier sine transform of $\frac{e^{-\alpha x}}{x}$**, where** $\alpha > 0$

9. a) Using Parseval's identity , evaluate $\int_0^\infty \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$

b) Find the Fourier Transform of f(x) given by

$$f(x) = \begin{cases} a-|x|, & if\ |x| < a \\ 0, & if\ |x| > a > 0 \end{cases}$$ **Hence deduce that** $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$.

10. a) Find the Fourier cosine transform of $f(x) = \begin{cases} 1-x^2, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$

Hence prove $\int_0^\infty \frac{\sin x - x\cos x}{x^3}\cos\left(\frac{x}{2}\right)dx = \frac{3\pi}{16}$

b) Find the Fourier cosine transform of $f(x) = \begin{cases} 1-x, 0 \le x \le 1 \\ 0, otherwise \end{cases}$ Hence find $\int_0^\infty \left(\frac{\sin x}{x}\right)^2 dx$

UNIT – V: Z – TRANSFOMS AND DIFFERENCE EQUATIONS

PART – A

1. **Define Z – Transform of the sequence** $\{f(n)\}$.
2. **Find the Z – Transform of unit step sequence.**
3. **Find** $Z(3^{n+2})$
4. **Find** $Z\left[\cos^2 \frac{n\pi}{2}\right]$.
5. **Find** $Z[n+2]$.
6. **Find** $Z\left[\frac{1}{n}\right]$.
7. **Find** $Z\left[\frac{a^n}{n!}\right]$.
8. **Find** $Z[a^n n]$.
9. **Prove that** $Z[a^n] = \frac{z}{z-a}$.
10. **Find** $Z\left[\frac{1}{n(n+1)}\right]$
11. **Find** $Z[n^2]$.
12. **Find** $Z\left[\frac{1}{(n+1)!}\right]$.
13. **State the Initial and Final value theorem.**
14. **Prove that $Z[f(n+1)] = zF(z) - zf(0)$.**
15. **Find the Z transform of $n(-1)^n$.**
16. **Find the inverse Z- transform of $\frac{z}{(z-1)(z-2)}$.**
17. **Form a difference equation by eliminating the arbitrary constants from** $y_n = A + B.2^n$.
18. **Solve $y_{n+1} - 2y_n = 0$ given that $y(0)=2$.**
19. **State Convolution theorem in Z – Transforms.**
20. **Find the inverse Z- transform of $\frac{z}{(z+1)^2}$**

PART – B

1. **a. Find** $(i) Z\left[r^n \cos n\theta\right]$, $(ii)\ Z\left[r^n \sin n\theta\right]$ $iii) Z(e^{-at}\cos bt)$

   b. Find the inverse Z -Transform of $\frac{z^3+3z}{(z-1)^2(z^2+1)}$.

2. **a. Find** $(i)\ Z\left[\sin\frac{n\pi}{2}\right]$, $(ii)\ Z\left[2^n \sin\frac{n\pi}{2}\right]$.

   b. Find $Z^{-1}\left(\frac{z}{z^2+7z+10}\right)$ **by convolution theorem.**

3. **a. Using Complex residue theorem evaluate** $Z^{-1}\left[\frac{9z^3}{(3z-1)^2(z-2)}\right]$.

   b. Solve $y_n + 3y_{n-1} - 4y_{n-2} = 0,\ n \geq 2.\ given\ \ that\ y(0)=3,\ y(1)=-2.$

4 **a. Solve by using Z-Transform** $y_{n+2} + 4y_{n+1} + 3y_n = 3^n,\ given\ that\ y_0 = 0\ and\ y_1 = 1.$

   b. Find the inverse Z – Transform of $\frac{z(z^2-z+2)}{(z+1)(z-1)^2}$.

5 **a. Find the Z-transform of** $\sin^3\left(\frac{n\pi}{6}\right)$.

   b. b. Solve $y(n+2)+6y(n+1)+9y(n)=2^n$ **given that** $y_0 = y_1 = 0$**using Z-transform.**

6 **a. Find the Z – Transform of** $f(n)=\frac{2n+3}{(n+1)(n+2)}$.

   b. Find i) $Z^{-1}\left[\frac{z^2}{(z-1)(z-3)}\right]$ **ii)** $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$ **iii)** $\frac{12z^2}{(3z-1)(4z+1)}$ **using convolution theorem.**

7 **a. If** $U(z) = \frac{2z^2+3z+12}{(z-1)^4}$ **, Find the values of** u_2 **and** u_3 **by residue method.**

   b. Form the difference equation from $Y(n) = (A+Bn)2^n$

8 **a. Find the Z transform of** $\frac{1}{(n+1)(n+2)}$ **and** $2^n \cos\frac{n\pi}{2}$.

   b. Solve the difference equation $y_{n+2} + 2y_{n+1} + y_n = n\ given y_0 = 0 = y_1$

9 **a)Solve the difference equation** $y(k+3)$ **- 3**$y(k+1)$**+2**$y(k)$**=0 with** $y(0)=4$**,**$y(1)=0$**and** $y(2)=8$**.**

   b) Find the inverse Z -Transform of $\frac{z^2}{(z+2)(z^2+4)}$ **by the method of Partial fraction**

10 **a. Solve the equation using Z – Transform** $y_{n+2} - 5y_{n+1} + 6y_n = 36$**, given that** $y(0) = y(1) = 0.$

   b)Using convolution theorem find the inverse Z transform of $\frac{8z^2}{(2z-1)(4z+1)}$