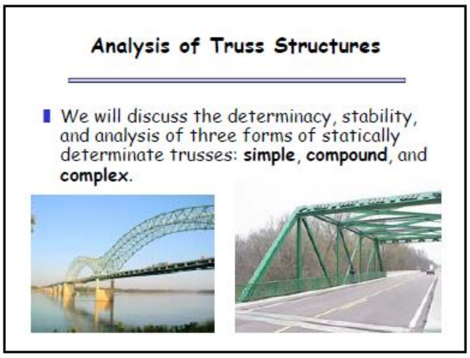
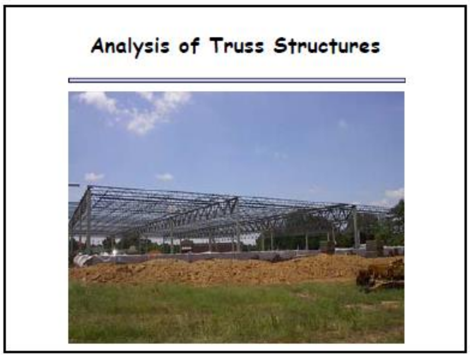
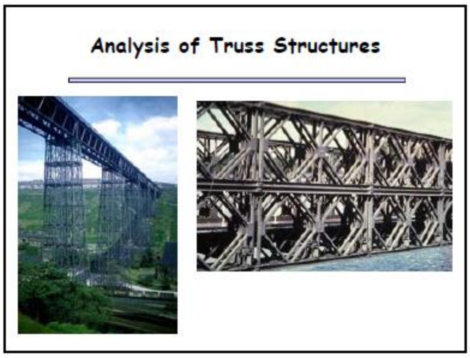
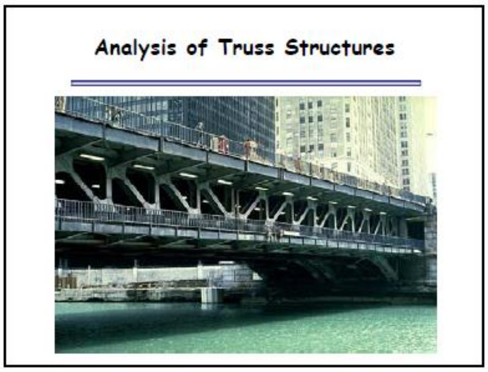
UNIT – I STATICALLY DETERMINATE STRUCTURES

Chap 1 Truss - Introduction







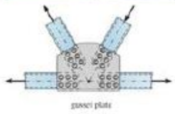


Analysis of Truss Structures

- Definition of a Truss
  - A **truss** is a structure composed of slender members joined together at their end points.
  - Planar trusses lie in a single plane.
  - Typically, the joint connections are formed by bolting or welding the end members together to a common plate, called a *gusset plate*.

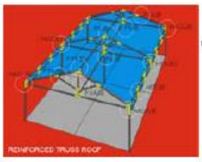
Analysis of Truss Structures

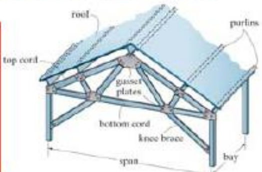
- Examples of gusset plates.



Analysis of Truss Structures

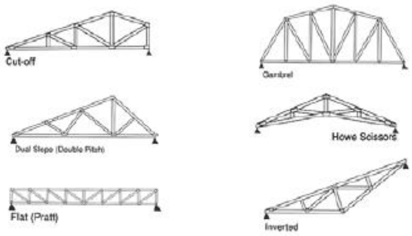
- Common Types of Trusses
  - **Roof trusses** - in general, the roof load is transmitted to the truss by a series of *purlins*. The roof truss along with its supporting columns is termed a *bent*. The space between bents is called a *bay*.





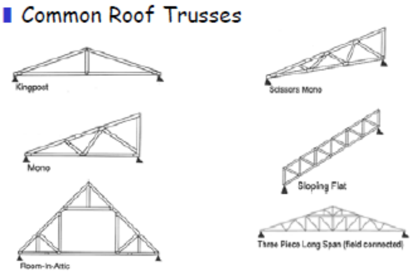
Analysis of Truss Structures

- Common Roof Trusses



Analysis of Truss Structures

- Common Roof Trusses

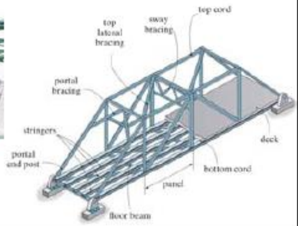


Analysis of Truss Structures

- Common Types of Trusses
  - **Bridge trusses** - the load is transmitted by the *deck* to a series of *stringers* and then to a set of *floor beams*.
  - The floor beams are supported by two parallel trusses.
  - The supporting trusses are connected top and bottom by *lateral bracing*.
  - Additional stability may be provided by *portal* and *sway* bracing

Analysis of Truss Structures

- Common Bridge Truss

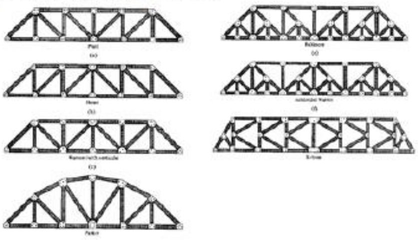


Analysis of Truss Structures

- Common Bridge Truss

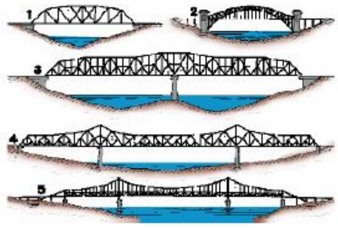
Analysis of Truss Structures

- Common Bridge Truss



Analysis of Truss Structures

- Common Bridge Truss



Analysis of Truss Structures

- Assumptions for Truss Design
  - To design both the members and connections of a truss, the *force* in each member for a given loading must be determined.
  - Two important assumptions are made in truss analysis:
    - *Truss members are connected by smooth pins*
    - *All loading is applied at the joints of the truss*

Analysis of Truss Structures

- ***Truss members are connected by smooth pins.***
  - The stress produced in these elements is called the *primary stress.*
  - The pin assumption is valid for bolted or welded connections if the members are concurrent.
  - However, since the connection does provide some rigidity, the bending introduced in the members is called *secondary stress.*
  - Secondary stress analysis is not commonly performed

Analysis of Truss Structures

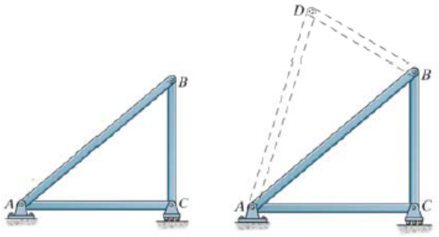
- *All loading is applied at the joints of the truss.*
  - Since the weight of each members is small compared to the member force, the member weight is often neglected.
  - However, when the member weight is considered, it is applied at the end of each member.
  - Because of these two assumptions, each truss member is a two-force member with either a compressive (C) or a tensile (T) axial force.
  - In general, compression members are bigger to help with instability due to buckling.

Classification of Coplanar Trusses

- Simple Truss
  - The simplest framework that is rigid or stable is a triangle.
  - Therefore, a simple truss is constructed starting with a basic triangular element and connecting two members to form additional elements.
  - As each additional element of two members is placed on a truss, the number of joints is increased by one.

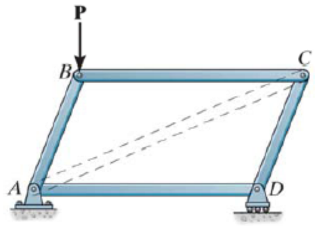
Classification of Coplanar Trusses

- Simple Truss



Classification of Coplanar Trusses

- Simple Truss



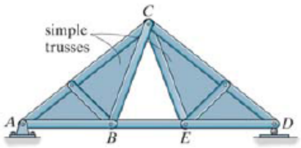
Classification of Coplanar Trusses

- Compound Truss
  - This truss is formed by connecting two or more simple trusses together.
  - This type of truss is often used for large spans.

Classification of Coplanar Trusses

- Compound Truss
  - There are three ways in which simple trusses may be connected to form a compound truss:

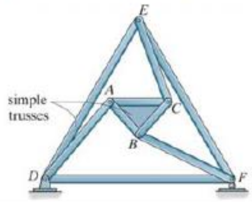
    1. Trusses may be connected by a common joint and bar.



Classification of Coplanar Trusses

- Compound Truss

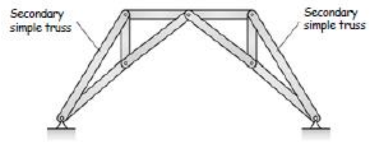
2. Trusses may be joined by three bars.



Classification of Coplanar Trusses

- Compound Truss

3. Trusses may be joined where bars of a large simple truss, called the *main truss*, have been substituted by simple trusses, called *secondary trusses*



Classification of Coplanar Trusses

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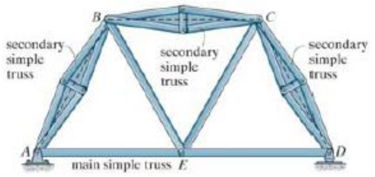
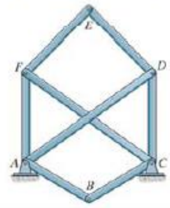


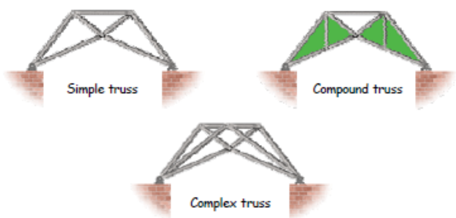
Classification of Coplanar Trusses

- Complex Truss
  - This is a truss that cannot be classified as being either simple or compound.



Classification of Coplanar Trusses

- Types of Trusses

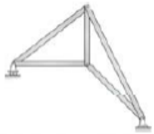


Determinacy of Coplanar Trusses

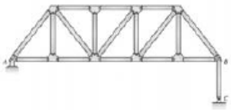
- Since all the elements of a truss are two-force members, the moment equilibrium is automatically satisfied.
- Therefore there are two equations of equilibrium for each joint, j, in a truss. If r is the number of reactions and b is the number of bar members in the truss, determinacy is obtained by

$b + r = 2j$	Determinate
$b + r > 2j$	Indeterminate

Determinacy of Coplanar Trusses

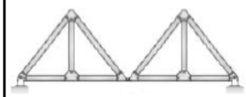


r = 3
b = 5 r+b=2j **determinate**
j = 4

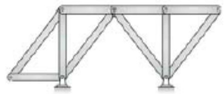


r = 4
b = 18 r+b=2j **determinate**
j = 11

Determinacy of Coplanar Trusses

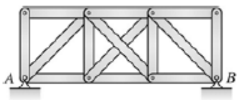


r = 4
b = 10 r+b=2j **determinate**
j = 7

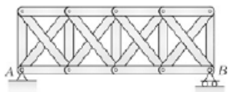


r = 4
b = 10 r+b=2j **determinate**
j = 7

Determinacy of Coplanar Trusses



r = 4
b = 14 r+b>2j **indeterminate**
j = 8



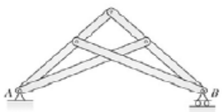
r = 3
b = 21 r+b>2j **indeterminate**
j = 10

Stability of Coplanar Trusses

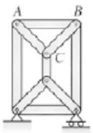
- If $b + r < 2j$, a truss will be ***unstable***, which means the structure will collapse since there are not enough reactions to constrain all the joints.
- A truss may also be unstable if $b + r \geq 2j$. In this case, stability will be determined by inspection

$b + r < 2j$	***Unstable***
$b + r \geq 2j$	***Unstable*** if reactions are concurrent, parallel, or collapsible mechanics

Stability of Coplanar Trusses



r = 3
b = 6 r+b<2j **unstable**
j = 5

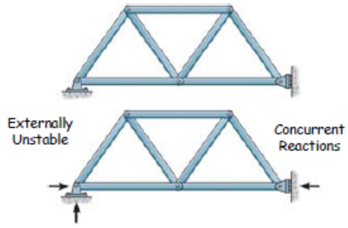


r = 3
b = 9 r+b=2j **unstable**
j = 6

Section ABC is supported by three parallel links

Stability of Coplanar Trusses

- **External stability** - a structure (truss) is externally unstable if its reactions are concurrent or parallel.



Chap 2 ANALSIS
Statically Determinate Structures

2.1 Introduction

- The purpose of this chapter is to review and reinforce the principle of static equilibrium within the context of some basic types of aircraft structures.
- It is important for a structural designer, in spite of – and aided by – digital computers, to develop a keen insight for predicting and visualizing load paths throughout a structure.
- The ability to do so largely depends on how well one has mastered the skills of stretching accurate free-body diagrams and properly applying the equilibrium equations to them, which will be one of our primary concerns here.
- For a structure in equilibrium, we must have (about any point P):

$$\sum F = 0 \quad and \quad \sum M_P = 0 \qquad [2.1.1]$$

- If there is a net imbalance of forces and moments on the structure, we know from dynamics that laws of of motion require that

$$\sum F = \dot{L}_{c.m.} \quad and \quad \sum M_{c.m.} = \dot{H}_{c.m.} \qquad [2.1.2]$$

where (Lc.m. dot) is the time rate of change of linear momentum of the structure's center of mass and (Hc.m. dot) is the time rate change of the structure's angular momentum about it center of mass.

- By rewriting Eqn. [2.1.1[in the form

$$\sum F + (-\dot{L}_{c.m.}) = 0 \quad and \quad \sum M_{c.m.} + (-\dot{H}_{c.m.}) = 0 \qquad [2.1.3]$$

which looks like Eqn. [2.1.1[with the (-Lc.m. dot) fictitious inertia force and (-Hc.m. dot) inertial couple applied at the center of mass (c.m.) – as though it were instantaneously in a state of "dynamic equilibrium" – is referred to as D'Alembert' s principle.

- Our focus in this chapter will be on statically determinate structures of the following types:

(1) pinned and rigid-jointed frames;

(2) stiffened shear webs (隔板);

(3) thin-walled beams and torque tubes

2.2 Plane Trusses

- A truss, also called a pin-jointed frame, is an idealized skeletal or “stick-like” structure composed of slender rod joined together by smooth pins at the joints (or nodes).
- The joints of a truss may be (1) pinned （訂住但可相互旋轉）or (2) welded or riveted together（全固定）.
- Ordinarily, external loads are applied only to the joints of a truss. A truss is a network of tensile and compressive forces (a two-force member), each having a known direction.
- The simplest plane truss consists of three rods pinned together to form a rigid triangle, as shown in Fig. [2.2.1]. If ‘j’ is the number of joints and ‘m’ is the

number of members, we see that for the triangular truss,

$$2j = m + 3 \qquad [2.2.1]$$

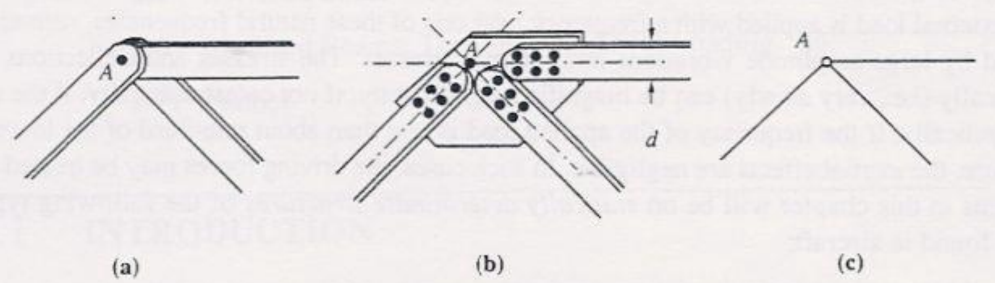


Figure 2.2.1 (a) Plane frame members held together by a single pin. (b) A riveted or bolted connection. (c) The idealized pin joint.

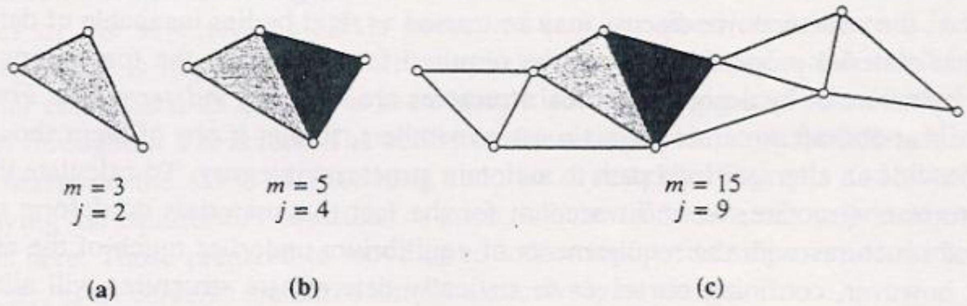


Figure 2.2.2 Stable trusses composed of triangular "building blocks."

- We can treat the entire truss as a free-body structure and use equilibrium equations to calculate the reactions at the support. If the number of equilibrium equations equals the number of unknown reactions, the truss is *externally statically determinate*. In two dimensions, we can write precisely three independent equilibrium equations for a rigid body. Therefore, if the number of unknown reactions exceeds three, the truss is externally statically indeterminate.

- A truss is *minimally stable* if it has the minimum number of rods required to support external loads and remain rigid. A truss composed of triangular subtrusses is minimally stable. If just one of the rods is removed, the truss will lose its rigidity and will become a mechanism. Rotation about one or more of the pins will occur, and the truss will collapse. A minimally stable plane truss is *internally statically determinate*. This means that we can calculate the forces in all of the rods if we are given the external loads at the joints.

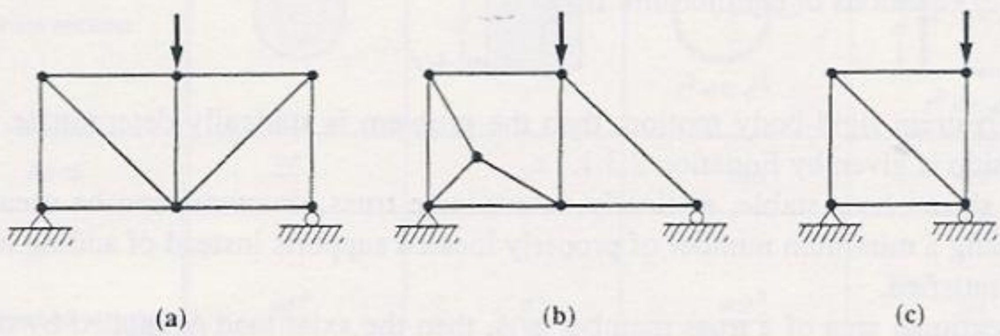


Figure 2.2.3 Examples of statically determinate, minimally stable trusses.

In Fig. 2.2.4, we know that 2j > m + 3; (a) j=5, m=6;(b) j=4, m=4;(c) j=5, m=6.

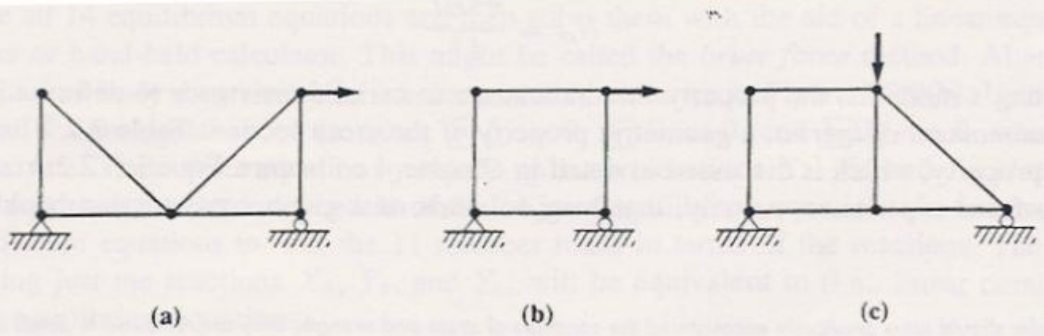


Figure 2.2.4 Examples of unstable trusses.

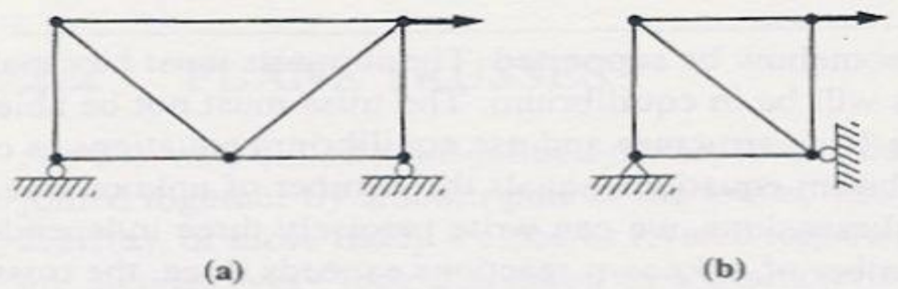


Figure 2.2.5 Examples of supports inadequate to restrain rigid-body translation (a) and rotation (b).

In Fig. 2.2.5 above, the truss is internally statically determinate trusses (2j = m + 3) ; The supports on the left truss cannot prevent rigid-body horizontal translation. One of the rollers should be replaced by a pin. The truss in part (b) of Fig. 2.2.5 seems to have the right number of supports. However, the roller at the wall cannot exert the force required to balance out the moment of the applied force about the pin. The roller should be on the floor to prevent rigid-body rotation around the pin.

compute forces in the rods and the reactions at the supports.)

- At the outset of a truss analysis, we know the applied loads.
- We can resolve each of the reactive loads into orthogonal components.
- Let r be the total number of reactions, and let j be the total number of joints. Pick any joint and isolate it as a free body.
- In two dimensions, we can write two equations of equilibrium for the joint. Doing this for every joint on the truss, we come up with $2j$ equations of equilibrium. If $2j = m+r$ and the the support restrain rigid-body motion, then the problem is statically determinate.

- Fig. 2.2.6 shows how stable, statically determinate truss structures can be created from unstable rod assemblies by adding a minimum number of properly located supports instead of adding members. In cases (a) and (b) in this Fig., Eqn. [2.2.2] (i.e., $2j = m+r$; r

is the number of reactions) is satisfied.

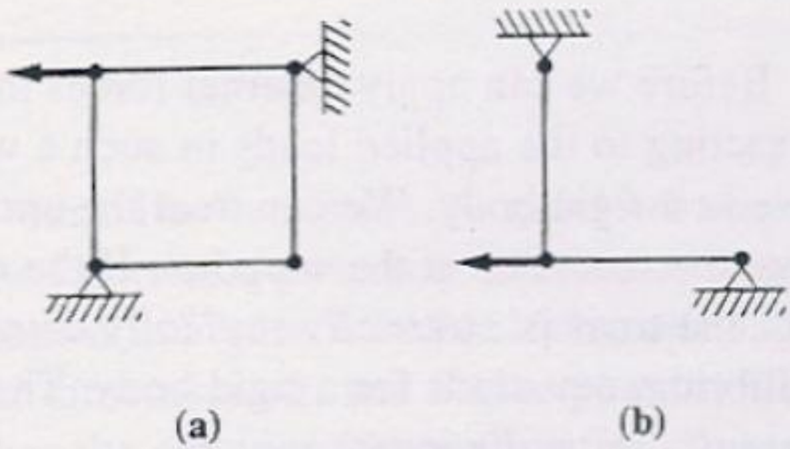


Figure 2.2.6 Unstable trusses made rigid by properly located additional supports.

- If the cross-sectional area of a truss member is A, then the axial load N applied by the smooth pins at each end produces a uniform normal stress (load intensity):

$$\sigma = \frac{N}{A} \qquad [2.2.3]$$

on cross sections throughout the bulk of the rod. To avoid mechanical failure (damage) of the rod, the value of the normal stress σ must remain within limits dictated by the strength of the material from which the rod is made.

- Furthermore, truss members that are in compression act like columns and may buckle, which is another form of failure to be avoided. In Chap 12, we show that a slender pin-supported rod of length L buckles at a critical load N_{cr} given by the classic Euler column formula:

$$N_{cr} = \frac{\pi^2 EI}{L^2} \qquad [2.2.4]$$

where E is Young's modulus, I is the area moment of inertia.

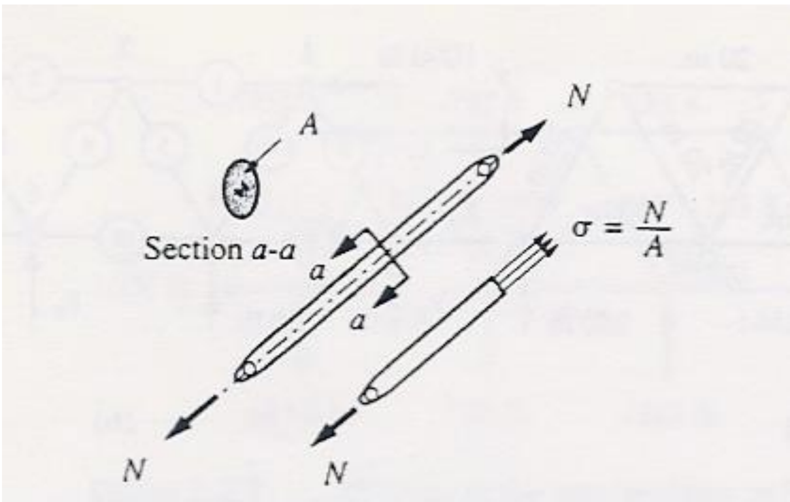
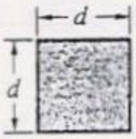
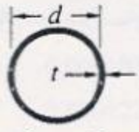
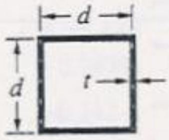


Figure 2.2.7 Uniform, uniaxial stress in a two-force rod element.

Table 2.2.1 Moments of inertia for some simple sections.

Cross section			 $t^2 << d^2$	 $t^2 << d^2$
Area	$\frac{\pi d^2}{4}$	d^2	πtd	$4td$
Moment of inertia	$\frac{\pi d^4}{16}$	$\frac{d^4}{12}$	$\frac{\pi t d^3}{8}$	$\frac{2td^3}{3}$

Eqn. (2.2.2) 2j = m + r , where r is the total number of reactions at supports.

Figure 2.2.8(a) shows a truss for which $j = 7$, $m = 11$, and $r = 3$, so that, according to Equation 2.2.2, the structure is statically determinate. To calculate the axial force in every member of the truss, we can employ the joint method. This procedure requires isolating each joint as a point in equilibrium under the action of the internal loads applied by the attached rods and the externally applied loads, possibly including those due to supports. Therefore, at each node, we obtain $\sum F_x = 0$ and $\sum F_y = 0$. The seven pairs of nodal equilibrium equations, as a system of 14 equations, will yield values for the 11 unknown member loads and the 3 unknown reactions.

We can write all 14 equilibrium equations and then solve them with the aid of a linear equation solver on a personal computer or hand-held calculator. This might be called the *brute force* method. Alternatively, we can begin by drawing a free-body diagram of the whole structure, as shown in Figure 2.2.8(b). Using that sketch, we write the three equilibrium equations of the truss, $\sum F_x = 0$, $\sum F_y = 0$, and $\sum M_P = 0$, where P is any point in the plane of the truss. These three equations yield the unknown reactions X_4, Y_4, and Y_6. Note that these three truss equilibrium equations are not independent of the 14 joint equilibrium equations. In fact, we can use any 11 of the joint equilibrium equations to find the 11 member loads in terms of the reactions. The remaining three equations, involving just the reactions X_4, Y_4, and Y_6, will be equivalent to (i.e., linear combinations of) the three overall truss equilibrium equations.

Thus, as an alternative to the brute force approach, we first obtain the reactions from Figure 2.2.8(b):

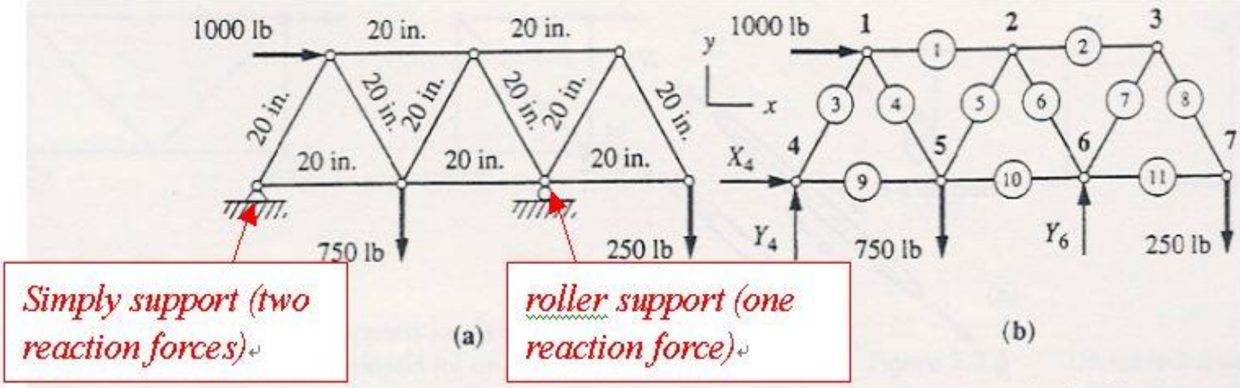


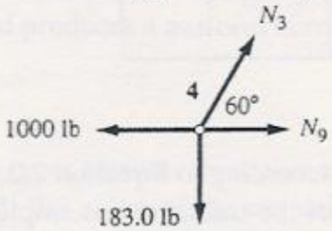
Figure 2.2.8 (a) Truss with loads and dimensions. (b) Truss as a free body, showing support reactions, and the chosen joint and member numbering scheme.

$$\sum F_x = 0: \quad X_4 + 1000 = 0 \quad X_4 = -1000 \text{ lb}$$

$$\sum M_4 = 0: \quad 40Y_6 - 20 \times 750 - 60 \times 250 - (20 \sin 60) \times 1000 = 0 \quad Y_6 = 1183 \text{ lb}$$

$$\sum F_y = 0: \quad Y_4 - 750 + 1183 - 250 = 0 \quad Y_4 = -183.0 \text{ lb}$$

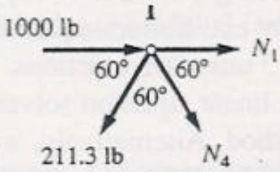
We then proceed from node to node, using the remaining 11 equilibrium equations to find the values of the member loads. If two unknown forces at most act at a joint, their values may be found by solving the two equilibrium equations at that point. By choosing the nodes judiciously, we can obtain all of the member loads "on the fly" as we work our way around the truss. In this case, joints 4 and 7 are candidate starting points for the solution process. Setting joint 4 in equilibrium yields the member forces N_3 and N_9:



$$\sum F_y = 0: \quad 0.8660N_3 - 183.0 = 0 \quad N_3 = 211.3 \text{ lb}$$

$$\sum F_x = 0: \quad 0.5000(211.3) + N_9 - 1000 = 0 \quad N_9 = 894.2 \text{ lb}$$

We then proceed to node 1, where the two member forces N_1 and N_4 are the unknowns:



$$\sum F_y = 0: \quad -0.8660(211.3) - 0.8660N_4 = 0 \quad N_4 = -211.3 \text{ lb}$$

$$\sum F_x = 0: \quad 1000 + N_1 - 0.5000(211.3) + 0.5000(-211.3) = 0 \quad N_1 = -788.8 \text{ lb}$$

In similar fashion, we obtain N_5 and N_{10} at node 5, N_2 and N_6 at node 2, and N_7 and N_{11} at node 6. At that

In similar fashion, we obtain N_5 and N_{10} at node 5, N_2 and N_6 at node 2, and N_7 and N_{11} at node 6. At that point, we will have used up all but one of the independent equilibrium equations. We may employ it at either joint 3 or joint 7 to solve for the remaining member force, N_8.

The results of this joint method of analysis are summarized in Figure 2.2.9. Since we assumed at the outset that all of the member loads were tensile (acting away from the joints), a minus sign means the member is in compression. Although we assumed that the structure was rigid, truss members do deform when loaded; therefore, those in compression are in jeopardy of buckling.

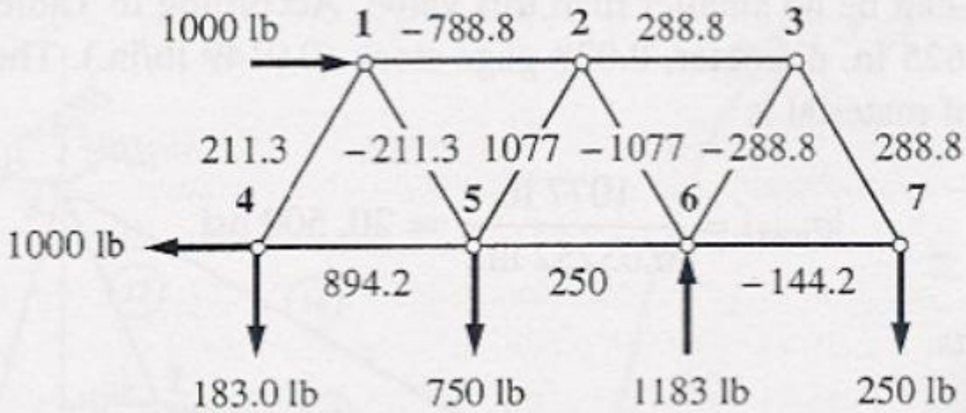


Figure 2.2.9 Solution of the truss problem in Figure 2.2.7.

Member loads are given in pounds (+ = tension, − = compression).

Example 2.2.1 All members of the truss in Figure 2.2.8 are to be fabricated from the same stock of thin-walled, round, steel tubing, the section properties of which are listed in Table 2.2.2. Select the lightest weight tubing for which the axial stress in any rod of the truss does not exceed 25,000 psi in tension or compression and the critical buckling load is not exceeded. For steel, $E = 30 \times 10^6$ psi.

Let us assume that the buckling criterion controls. Since all members of this truss have the same length, we examine Figure 2.2.9 for the largest compressive load, which is 1077 lb in rod 6. This force also happens to be the largest tensile load in the truss (rod 5). Therefore, sizing just member 6 will do for the entire structure. (Reality is rarely this simple!) From Equation 2.2.4, we have for rod 6,

$$1077 = \frac{\pi^2(30 \times 10^6)I}{20^2}$$

so that, for the tube to be on the verge of buckling,

$$I = 0.001455 \text{ in.}^4$$

The area moment of inertia we select can be no smaller than this value. According to Table 2.2.2, the lightest weight tubing for which $I > 0.001455$ in.^{4} is the 0.625 in. diameter, 0.028 gage stock (0.0149 lb/in.). The maximum tensile and compressive stress in the truss for this choice of material is

$$|\sigma_{\max}| = \frac{1077 \text{ lb}}{0.05252 \text{ in.}^2} = 20,500 \text{ psi}$$

This is well within the 25,000 psi limits.

An alternative to the joint method of plane truss analysis is the *method of sections*. Because this method avoids having to work through the entire structure, it is the method of choice whenever we need to find the force in just a few members of a statically determinate truss. After identifying the member of interest, we section the truss into two free bodies so as to expose the force in that member. We then write the equations of equilibrium for the free body on either side of the section and solve them for the unknown force. For example, to find the member forces in the center bay of the cantilever truss in Figure 2.2.10(a), make an imaginary vertical cut through that bay and isolate the portion of the truss to the left of the cut so as to form the free-body diagram in Figure 2.2.10(b). The three equations of equilibrium for this free body yield the three unknown member forces:

$$\sum F_y = 0 \quad \Rightarrow \quad N_{5,8} = \sqrt{2}P$$
$$\sum M_5 = 0 \quad \Rightarrow \quad N_{6,8} = 2P$$
$$\sum F_x = 0 \quad \Rightarrow \quad N_{5,7} = -3P$$

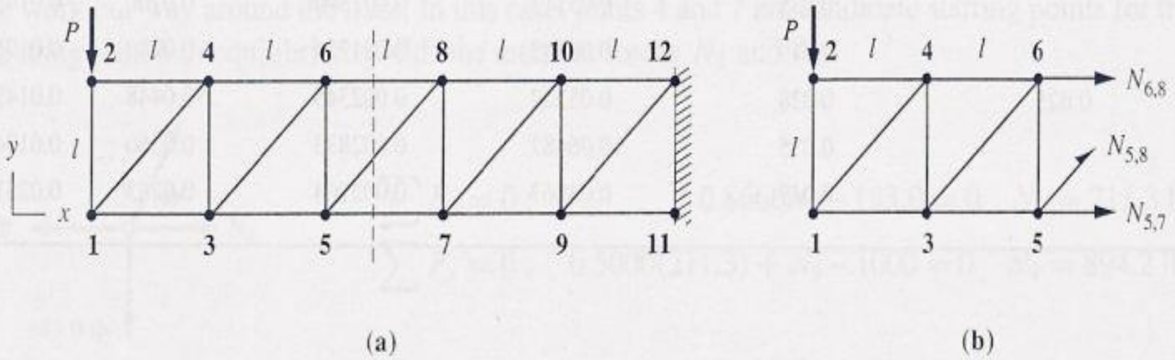
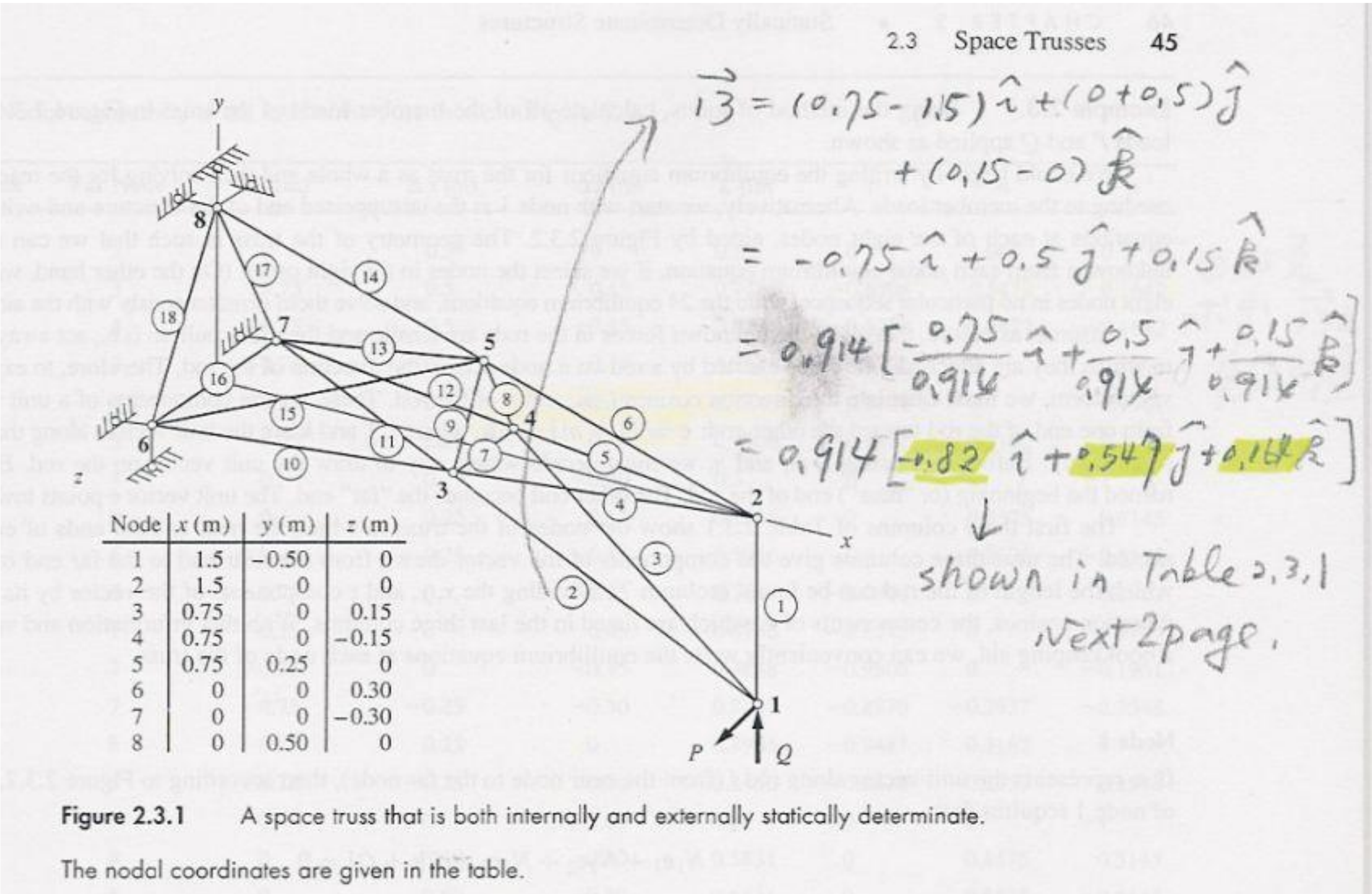


Figure 2.2.10 (a) Cantilevered truss with a transverse section through the center bay. (b) Free-body diagram to the left of the cut, revealing the member forces in that bay.

2.3 Space Trusses

- Just as for a plane truss, a space truss (3-D) must be supported in such a way that rigid-body translation and rotation are prohibited.



Node	x (m)	y (m)	z (m)
1	1.5	−0.50	0
2	1.5	0	0
3	0.75	0	0.15
4	0.75	0	−0.15
5	0.75	0.25	0
6	0	0	0.30
7	0	0	−0.30
8	0	0.50	0

Figure 2.3.1 A space truss that is both internally and externally statically determinate. The nodal coordinates are given in the table.

- In above cantilevered space truss, the supports at the wall are represented by short links (Pt. 8 : 3 d.o.f. fixed; Pt. 6 : 2 d.o.f. fixed; Pt. 7 : 1 d.o.f. fixed).
  - The truss is externally statically determinate

because the six forces exerted by the links on the truss can be found in terms of the applied loads P and Q by means of six equations of equilibrium (i.e., $\sum F = 0$ and $\sum M = 0$).

- Furthermore, the truss is internally statically determinate since the number of rods (18) plus the number of reactions (6) equals 24 = total number of joint equilibrium equations (8 nodes, 3 equations per node).

- The truss shown in Fig. 2.3.1 is minimally stable, and note that in three-dimensions, Eqn. (2.2.2) is replaced by

$$3j = m + r \qquad [2.3.1]$$

where j is the no. of joint, m is the no. of truss member, and r is the no. of reactions (equals six for 3-D problem).

Example 2.3.1 Using the method of joints, calculate all of the member loads of the truss in Figure 2.3.1 in terms of the loads P and Q applied as shown.

We could begin by writing the equilibrium equations for the truss as a whole and then solving for the reactions before proceeding to the member loads. Alternatively, we start with node 1 at the unsupported end of the structure and write the equilibrium equations at each of the eight nodes, aided by Figure 2.3.2. The geometry of the truss is such that we can find three of the unknowns from each nodal equilibrium equation, if we select the nodes in the right order. (On the other hand, we could select the eight nodes in no particular sequence, write the 24 equilibrium equations, and solve them simultaneously with the aid of a computer.)

Assume, as before, that all of the unknown forces in the rods are tensile and therefore pull on (i.e., act away from) the nodes to which they are attached. The force exerted by a rod on a node acts in the direction of the rod. Therefore, to express the force in vector form, we must calculate the direction cosines l, m, and n of the rod. These are the components of a unit vector $\mathbf{e}$ pointing from one end of the rod toward the other end: $\mathbf{e} = l\mathbf{i} + m\mathbf{j} + n\mathbf{k}$, where $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are the unit vectors along the x, y, and z axes, respectively. Before calculating l, m, and n, we must decide which way to draw the unit vector on the rod. Either end can be named the beginning (or "near") end of the rod. The other end becomes the "far" end. The unit vector $\mathbf{e}$ points toward the far end.

The first three columns of Table 2.3.1 show the nodes of the truss to which the near and far ends of each rod are connected. The next three columns give the components of the vector drawn from the near end to the far end of each rod, from which the length of the rod can be found (column 7). Dividing the x, y, and z components of the vector by its length yields its direction cosines, the components of $\mathbf{e}$, which are listed in the last three columns. With this information and vector notation as a bookkeeping aid, we can conveniently write the equilibrium equations at each node of the truss.

Node 1

If $\mathbf{e}_i$ represents the unit vector along rod i (from the near node to the far node), then according to Figure 2.3.2, the equilibrium of node 1 requires that

$$N_1\mathbf{e}_1 + N_2\mathbf{e}_2 + N_3\mathbf{e}_3 + P\mathbf{k} + Q\mathbf{j} = 0 \qquad \text{[a]}$$

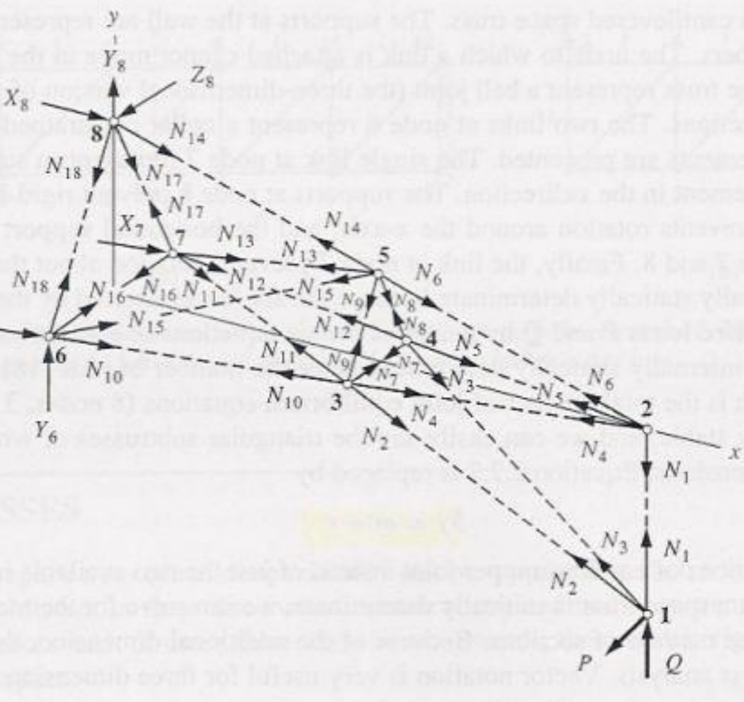


Figure 2.3.2 Free-body diagrams of nodes 1 through 5 of the truss in Figure 2.3.1.

Table 2.3.1 Nodal connectivity of the rods in the truss shown in Figure 2.3.1.

Rod	Near Node	Far Node	Δx (m)	Δy (m)	Δz (m)	L (m)	l	m	n
1	1	2	0	0.5	0	0.5	0	1.000	0
2	1	3	−0.75	0.5	0.15	0.9138	−0.8208	0.5472	0.1642
3	1	4	−0.75	0.5	−0.15	0.9138	−0.8208	0.5472	−0.1642
4	2	3	−0.75	0	0.15	0.7648	−0.9806	0	0.1961
5	2	4	−0.75	0	−0.15	0.7648	−0.9806	0	−0.1961
6	2	5	−0.75	0.25	0	0.7906	−0.9487	0.3162	0
7	3	4	0	0	−0.30	0.3	0	0	−1
8	4	5	0	0.25	0.15	0.2916	0	0.8575	0.5145
9	3	5	0	0.25	−0.15	0.2916	0	0.8575	−0.5145
10	3	6	−0.75	0	0.15	0.7648	−0.9806	0	0.1961
11	3	7	−0.75	0	−0.45	0.8746	−0.8575	0	−0.5145
12	4	7	−0.75	0	−0.15	0.7648	−0.9806	0	−0.1961
13	5	7	−0.75	−0.25	−0.30	0.8456	−0.8870	−0.2957	−0.3548
14	5	8	−0.75	0.25	0	0.7906	−0.9487	0.3162	0
15	5	6	−0.75	−0.25	0.30	0.8456	−0.8870	−0.2957	0.3548
16	6	7	0	0	−0.60	0.6	0	0	−1
17	7	8	0	0.50	0.30	0.5831	0	0.8575	0.5145
18	6	8	0	0.50	−0.30	0.5831	0	0.8575	−0.5145

Since the components of $\mathbf{e}_i$ are the direction cosines of rod i, we can go to Table 2.3.1 to find

$$\mathbf{e}_1 = \mathbf{j} \qquad \mathbf{e}_2 = -0.8208\mathbf{i} + 0.5472\mathbf{j} + 0.1642\mathbf{k} \qquad \mathbf{e}_3 = -0.8208\mathbf{i} + 0.5472\mathbf{j} - 0.1642\mathbf{k}$$

Substituting these unit vectors into Equation (a) and collecting terms, we obtain

$$(-0.8208N_2 - 0.8208N_3)\mathbf{i} + (N_1 + 0.5472N_2 + 0.5472N_3 + Q)\mathbf{j} + (0.1642N_2 - 0.1642N_3 + P)\mathbf{k} = 0$$

The three scalar equilibrium equations at node 1 are obtained by setting the x, y, and z components of this vector equation equal to zero:

$$-0.8208N_2 - 0.8208N_3 = 0$$
$$N_1 + 0.5472N_2 + 0.5472N_3 = -Q$$
$$0.1642N_2 - 0.1642N_3 = -P$$

The solution of this system of equations is

$$N_1 = -Q \qquad N_2 = -3.046P \qquad N_3 = 3.046P$$

$$N_1 = -Q \qquad N_2 = -3.046P \qquad N_3 = 3.046P$$

Node 2

We use the free-body diagram in Figure 2.3.2 to write

$$N_1(-\mathbf{e}_1) + N_4\mathbf{e}_4 + N_5\mathbf{e}_5 + N_6\mathbf{e}_6 = 0$$

Since N_1 acts away from node 2, in the direction opposite to that assumed for $\mathbf{e}_1$, this explains the minus sign in the first term. As before, we use the information in the table to express the unit vectors in terms of their $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components.

Substituting those expressions into this equation, we get three scalar equations, as follows:

$$-0.9806N_4 - 0.9806N_5 - 0.9487N_6 = 0$$

$$N_1 + 0.3162N_5 = 0$$

$$0.1961N_4 - 0.1961N_5 = 0$$

We already found that $N_1 = -Q$. Therefore, from this set of equilibrium equations, we readily obtain

$$N_4 = 1.530Q \quad N_5 = 1.530Q \quad N_6 = -3.162Q$$

Node 4

We go to this node next since only three unknown forces, N_7, N_8, and N_{12}, act on it, whereas there are four unknowns at node 3 and five at node 5. The equilibrium equation for node 4 is

$$N_3(-\mathbf{e}_3) + N_5(-\mathbf{e}_5) + N_7(-\mathbf{e}_7) + N_8\mathbf{e}_8 + N_{12}\mathbf{e}_{12} = 0$$

Substituting $N_3 = 3.046P$ and $N_5 = 1.530Q$, as just determined, and using the data in Table 2.3.1, we get, after simplification,

$$-0.9806N_{12} = -2.5P - 1.5Q$$

$$0.8575N_8 = 1.667P$$

$$N_7 + 0.5145N_8 - 0.1961N_{12} = -0.5P - 0.3Q$$

These immediately yield N_7, N_8, and N_{12} (see Table 2.3.2).

Node 3

Referring to Figure 2.3.2, we write

$$N_2(-\mathbf{e}_2) + N_4(-\mathbf{e}_4) + N_7\mathbf{e}_7 + N_9\mathbf{e}_9 + N_{10}\mathbf{e}_{10} + N_{11}\mathbf{e}_{11} = 0$$

Writing out the unit vectors using Table 2.3.1 and using the values of N_2, N_4, and N_7 just obtained yields three equations in the three unknowns N_9, N_{10}, and N_{11}.

Continuing in this fashion, we proceed next to node 5 to find N_{13}, N_{14}, and N_{15}, and then to node 7, which yields N_{16}, N_{17}, and the reaction $X_7 = -2.5P - 1.5Q$. At node 6, we then obtain N_{18} and the reactions $X_6 = 2.5P - 1.5Q$ and $Y_6 = -3.333P$. Finally, at node 8, we solve for the last of the 24 original unknowns, namely the reaction components:

$$X_8 = 3Q \quad Y_8 = 3.333P - Q \quad Z_8 = -P$$

The internal loads throughout the structure are summarized in Table 2.3.2.

Table 2.3.2 Axial loads in the members of the space truss in Figure 2.3.1.

Member	Axial Force
1	Q
2	$-3.046P$
3	$3.046P$
4	$1.530Q$
5	$1.530Q$
6	$-3.162Q$
7	$-P$
8	$1.944P$

Table 2.3.2 Concluded

Member	Axial Force
9	21.944P
10	$-5.099P+1.530P$
11	$2.916P$
12	$2.540P+1.530Q$
13	$-2.819P$
14	$-3.162Q$
15	$2.819P$
16	$-1.500P-0.3000Q$
17	$0.9718P$
18	2.916P

Figure 2.3.3(a) shows another minimally stable, statically determinate space truss. It is supported against rigid-body translations and rotations. To find the forces in the members in terms of the applied 1000 lb load, we must write the equilibrium equations at each node. This will give us 24 equations for the 18 member loads and the 6 reactions. Unlike the previous example, on this truss, there is no joint with fewer than four rods framing into it. Therefore, the 24 equations do not uncouple into smaller, independent groups of equations that will yield the unknowns as we proceed from node to node around the structure. Instead they must be solved simultaneously to find the forces shown in Figure 2.3.3(b).

The procedure for solving determinate space trusses is straightforward, but extremely lengthy and tedious. Yet, the joint method is easy to program for the most modest personal computer. However, such a program, by itself, would have limited application in "real world" situations, where trusses are likely to be statically indeterminate. In fact, it is difficult to design space trusses that are not statically indeterminate. For example, suppose we simply add

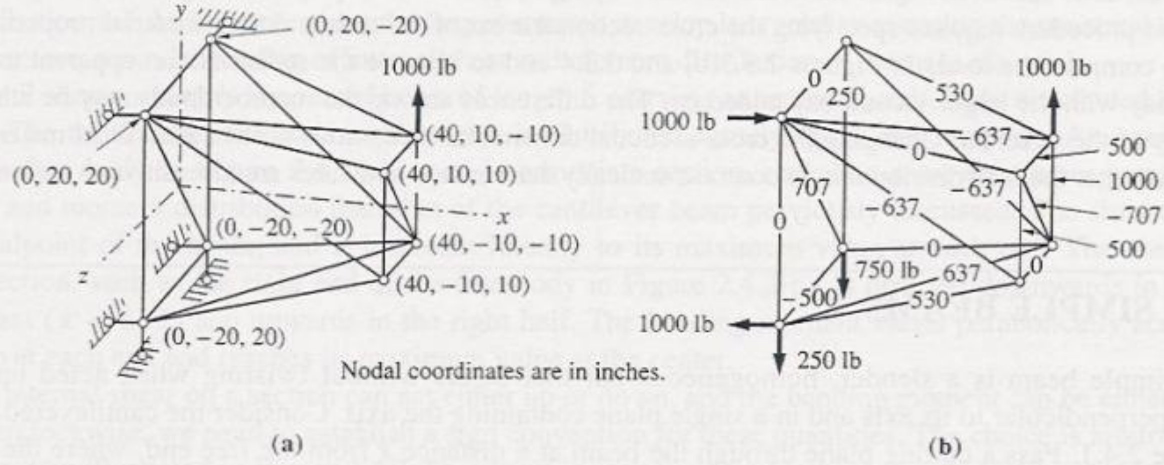


Figure 2.3.3 (a) Statically determinate space truss. (b) The reactions and member loads (in pounds).

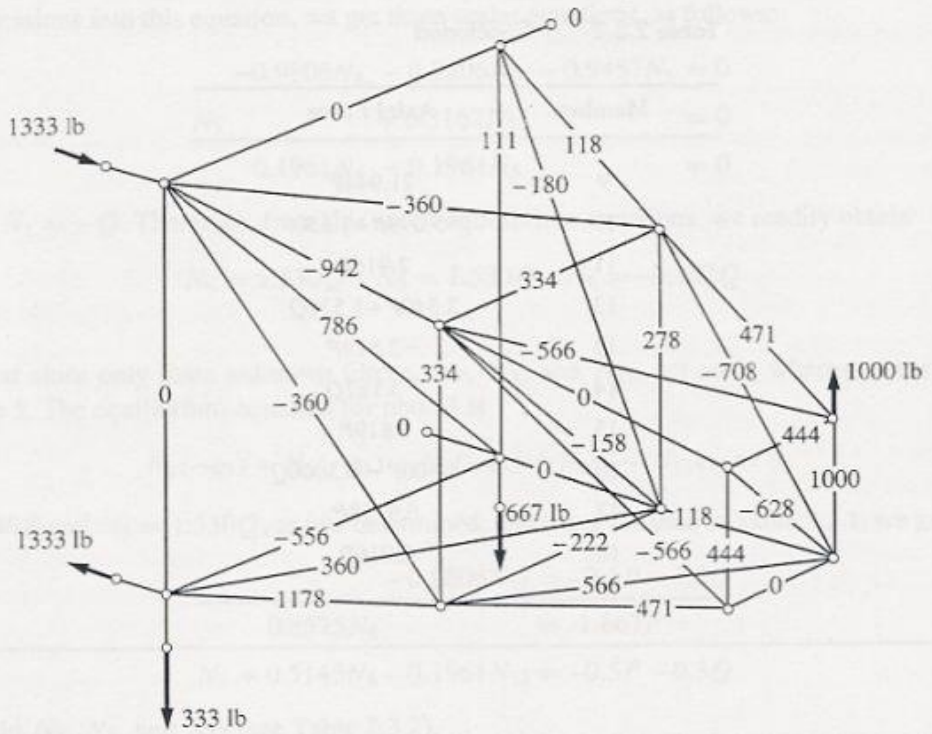


Figure 2.3.4 Member forces (in pounds) and the reactions of a two-bay space truss similar to the one in Figure 2.3.3.

All rods have the same cross sectional area and the same material properties.

another identical, tapered, minimally stable bay to the statically determinate truss in Figure 2.3.3(a) and support it in the same way, so that the number of reactions remains the same. The result is shown in Figure 2.3.4. The new bay adds 13 members to those already present. It also adds four nodes, which means we have only 12 additional joint equilibrium equations. For this expanded truss, we are one equation short of matching the total number of unknown member forces and reactions. If we add a third bay, we fall two equations behind, and so on. The degree of static indeterminacy increases with each additional bay.

The means of dealing with statically indeterminate trusses are explored in Chapters 7 and 10. The member forces and the reactions shown in Figure 2.3.4 are obtained by a procedure that requires dealing with the deformation of the truss. This procedure requires specifying the cross-sectional areas of the rods and their material properties. It is interesting to compare the loads in Figures 2.3.3(b) and 2.3.4 and to observe the redistribution apparent in the original, smaller bay with the larger second bay added on. The differences among the member loads may be attributed to the flexibility of the structure. Changing the cross-sectional dimensions of the rods or their individual material properties will further alter the load distribution. In contrast, statically determinate structures are insensitive to such modifications.